Math 310 Fall, 2020
Krantz

SECOND MIDTERM EXAM

General Instructions: Read the statement of each problem carefully. If you want full credit on a problem then you must show your work. If you only write the answer then you will *not* receive full credit.

Be sure to ask questions if anything is unclear.

(6 points) **1.** Let $S = \{2, 3, 4, 6\}$ and $T = \{1, 3, 5, 7\}$. What is $S \cup T$? What is $S \cap T$? What is $S \setminus T$?

(6 points) **2.** Let $S = \{2, 4, 6\}$ and $T = \{a, b, c, d\}$. What is $S \times T$? What is $T \times S$?

(8 points) **3.** Draw a Venn diagram to illustrate the identity

$$S \setminus (T \cup U) = (S \setminus T) \cap (S \setminus U) \, .$$

(6 points) **4.** What is the power set of $\{3, \alpha, x\}$?

(9 points) **5.** Which of these functions is one-to-one? Which is onto (give a brief reason for each answer)?

(a) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^3 + x$

(b) $g : \mathbb{N} \to \mathbb{N}$ $\quad g(n) = n(n+1)$

(c) $h : \mathbb{R} \to \mathbb{R}$ $\quad h(x) = x \sin x$

(9 points) **6.** Which of these sets is countable and which uncountable (give a brief reason for each answer)?

(a) $\mathbb{C} \times \mathbb{R}$

(b) $\mathbb{Z} \times \mathbb{N}$

(c) $\mathbb{Z} \times \mathbb{C}$

(6 points) **7.** Calculate the inverse of the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x & \text{if} \quad x \leq 0 \\ x^2 & \text{if} \quad x > 0\,. \end{cases}$$

(6 points) **8.** Prove that the collection of S of irrational numbers is uncountable.

(8 points) **9.** Explain why the product of a countable set and an uncountable set is uncountable.

(6 points) **10.** Explain why the union of a countable set and an uncountable set is uncountable.

(8 points) **11.** Prove that addition in the integers is well defined. You should use the actual, *rigorous* definition of the integers (in terms of ordered pairs of natural numbers) to do this problem.

(8 points) **12.** What is the multiplicative inverse of the complex number $2 - 3i$?

(8 points) **13.** Find a square root in the quaternions of the quaternion $4 \cdot \mathbf{1} + 4 \cdot \mathbf{k}$.

(6 points) **14.** Find all cube roots of the complex number i.