SECOND MIDTERM EXAM

General Instructions: Read the statement of each problem carefully. If you want full credit on a problem then you must show your work. If you only write the answer then you will not receive full credit.

Be sure to ask questions if anything is unclear.

(6 points) 1. Let $S = \{2, 3, 4, 6\}$ and $T = \{1, 3, 5, 7\}$. What is $S \cup T$? What is $S \cap T$? What is $S \setminus T$?

(6 points) 2. Let $S = \{2, 4, 6\}$ and $T = \{a, b, c, d\}$. What is $S \times T$? What is $T \times S$?

(8 points) 3. Draw a Venn diagram to illustrate the identity

$$S \setminus (T \cup U) = (S \setminus T) \cap (S \setminus U).$$

(6 points) 4. What is the power set of $\{3, \alpha, x\}$?

(9 points) 5. Which of these functions is one-to-one? Which is onto (give a brief reason for each answer)?

(a) $f : \mathbb{R} \to \mathbb{R}$ \quad $f(x) = x^3 + x$
(b) $g : \mathbb{N} \to \mathbb{N}$ \quad $g(n) = n(n + 1)$
(c) $h : \mathbb{R} \to \mathbb{R}$ \quad $h(x) = x \sin x$

(9 points) 6. Which of these sets is countable and which uncountable (give a brief reason for each answer)?

(a) $\mathbb{C} \times \mathbb{R}$
(b) \(\mathbb{Z} \times \mathbb{N}\)
(c) \(\mathbb{Z} \times \mathbb{C}\)

(6 points) 7. Calculate the inverse of the function \(f : \mathbb{R} \to \mathbb{R}\) given by

\[
f(x) = \begin{cases} 
  x & \text{if } x \leq 0 \\
  x^2 & \text{if } x > 0
\end{cases}
\]

(6 points) 8. Prove that the collection of \(S\) of irrational numbers is uncountable.

(8 points) 9. Explain why the product of a countable set and an uncountable set is uncountable.

(6 points) 10. Explain why the union of a countable set and an uncountable set is uncountable.

(8 points) 11. Prove that addition in the integers is well defined. You should use the actual, rigorous definition of the integers (in terms of ordered pairs of natural numbers) to do this problem.

(8 points) 12. What is the multiplicative inverse of the complex number \(2 - 3i\)?

(8 points) 13. Find a square root in the quaternions of the quaternion \(4 \cdot 1 + 4 \cdot k\).

(6 points) 14. Find all cube roots of the complex number \(i\).