Math 310 Fall, 2020
Krantz

PRACTICE EXAM FOR SECOND MIDTERM

1. Let $S = \{3, 4, 6\}$ and $T = \{3, 5\}$. What is $S \cup T$? What is $S \cap T$? What is $S \setminus T$?

2. Let $S = \{2, 4\}$ and $T = \{a, b, d\}$. What is $S \times T$? What is $T \times S$?

3. Draw two Venn diagrams to illustrate the identity

$$(T \cup U) \setminus S = (T \setminus S) \cup (U \setminus S)\,.$$

4. What is the power set of $\{\lambda, A, 2\}$?

5. Which of these functions is one-to-one? Which is onto (give a brief reason for each answer)?

(a) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2 + x$

(b) $g : \mathbb{N} \to \mathbb{N}$ $\quad g(n) = n(n + 3)$

(c) $h : \mathbb{R} \to \mathbb{R}$ $\quad h(x) = x \cos x$

6. Which of these sets is countable and which uncountable (give a brief reason for each answer)?

(a) $\mathbb{C} \times \mathbb{C}$

(b) $\mathbb{Z} \times \mathbb{Q}$

(c) $\mathbb{N} \times \mathbb{R}$

7. Calculate the inverse of the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^3 & \text{if} \quad x \le 0 \\ x & \text{if} \quad x > 0\,. \end{cases}$$

8. Prove that the collection of S of rational numbers with denominator 7 is countable.

9. Explain why the product of an uncountable set and a countable set is uncountable.

10. Explain why the union of a uncountable set and a countable set is uncountable.

11. Prove that subtraction in the integers is well defined. You should use the actual, *rigorous* definition of the integers in terms of ordered pairs of natural numbers to do this problem. Also use the rigorous definition of subtraction provided in the book and the lectures.

12. What is the multiplicative inverse of the complex number $-i$?

13. Find a square root in the quaternions of the quaterion $2 \cdot \mathbf{1} + 2 \cdot \mathbf{j}$.

14. Find all square roots of the complex number $1 - i$.