

PRACTICE FIRST MIDTERM

General Instructions: Read the statement of each problem carefully. On each problem you should show your work. If you only write the answer then you will not receive full credit.

Be sure to ask questions if anything is unclear. This exam is worth 100 points.

(14 points) 1. Calculate the multiplicative inverse of $z = 13 + 3i$.

(14 points) 2. Calculate this derivative:

$$\frac{\partial}{\partial z} (\bar{z} \cdot z^2 + z \sin z) .$$

(16 points) 3. Show that the function $u(x, y) = x + y$ is harmonic. Find the real-valued harmonic conjugate function v so that $u + iv$ is holomorphic.

(14 points) 4. Explain why every holomorphic function is harmonic but the converse is not true.

(14 points) 5. Give an example of a holomorphic function in the unit disc that has infinitely many zeros but is not identically zero.

(14 points) 6. The Cauchy integral theorem does not apply to the function $f(z) = x$. Explain why.

(14 points) 7. The Cauchy integral formula does not apply to the function $f(z) = x$. Explain why.