Math 416   Spring, 2023
Krantz

PRACTICE FIRST MIDTERM

(14 points)   **1.** Calculate the multiplicative inverse of $z = 13 + 3i$.

(14 points)   **2.** Calculate this derivative:

$$\frac{\partial}{\partial z} \left(\bar{z} \cdot z^2 + z \sin z \right) .$$

(16 points)   **3.** Show that the function $u(x, y) = x + y$ is harmonic. Find the real-valued harmonic conjugate function v so that $u + iv$ is holomorphic.

(14 points)   **4.** Explain why every holomorphic function is harmonic but the converse is not true.

(14 points)   **5.** Give an example of a holomorphic function in the unit disc that has infinitely many zeros but is not identically zero.

(14 points)   **6.** The Cauchy integral theorem does not apply to the function $f(z) = x$. Explain why.

(14 points)   **7.** The Cauchy integral formula does not apply to the function $f(z) = x$. Explain why.