SOLUTIONS TO PRACTICE MIDTERM ONE

1. $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{|z|^2} = \frac{13-3i}{169+9} = \frac{13}{178} - \frac{3}{178} i$

2. $\frac{\partial}{\partial z}\left(\bar{z} \cdot z^2 + z \sin z\right) = 2\bar{z} \cdot z + \sin z + z \cos z$

3. $\Delta(x+y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(x+y) = 0 + 0 = 0$

$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = -1$

$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = 1$

$\therefore v(x,y) = y + \varphi(x)$

$-1 = \frac{\partial v}{\partial x} = \varphi'(x) \Rightarrow \varphi(x) = -x.$

So $v(x,y) = y - x$

The holo fcn. is

$$h(x,y) = (x+y) + i(y-x) = (1-i)z$$

4. If f is holo, then $\frac{\partial}{\partial \bar{z}} f = 0$. So

$$\frac{\partial}{\partial z}\frac{\partial}{\partial \bar{z}} f = 0$$

$$\frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) f = 0$$

$$\frac{1}{4}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f = 0$$

$$\Delta f = 0.$$

5. $f(z) = \sin \frac{1}{1-z}$

This function vanishes when

$$\frac{1}{1-z} = k\pi, \quad k \in \mathbb{Z}.$$

or

$$1 = k\pi - k\pi z$$

$$z = \frac{k\pi - 1}{k\pi} = 1 - \frac{1}{k\pi}.$$

6. $f(z) = x$ is not holomorphic. One can also calculate that

$$\oint_\gamma f(z)\,dz \neq 0.$$

7. $f(z) = x$ is not holomorphic. One can also calculate (for $\zeta = \xi + i\eta$) that

$$\frac{1}{2\pi i} \oint_\gamma \frac{\xi}{\zeta - z}\,d\zeta \neq x.$$