

PRACTICE SECOND MIDTERM

1. Identify the type of singularity at 0 for each of these functions:

(a) $f(z) = \frac{\sin z}{z^3}$

(b) $g(z) = e^{1/z^2}$

(c) $h(z) = \frac{z}{\cos z}$

(d) $k(z) = \frac{\cot z}{\sin z}$

2. Calculate the Laurent series for the function

$$f(z) = \frac{e^z}{z^2}.$$

3. Calculate the explicit value of the integral

$$\oint_{|z|=5} \frac{dz}{(z-1)(z+4)}.$$

4. Use the calculus of residues to calculate the explicit value of the integral

$$\int_{-\infty}^{\infty} \frac{dx}{4+x^2}.$$

5. What does the Schwarz lemma say? [**Hint:** Be sure to include *all parts* of the lemma.]
6. Give a complete description of all the conformal self-maps of the unit disc.
7. Use Rouché's theorem to determine the number of zeros of the function $f(z) = -z^2 + 5z + 2$ in the unit disc.

8. What does Hurwitz's theorem say about the sequence of holomorphic functions $\{e^{z^2/j^3}\}$?
9. What does the boundary maximum modulus principle say?
10. What is the winding number of the curve $\gamma(t) = 3e^{-8it}$, $0 \leq t \leq \pi$ about the origin?