PRACTICE EXAM FOR SECOND MIDTERM

(10 points) **1.** What do the Cauchy estimates say?

(10 points) **2.** Use the Cauchy estimates to prove Liouville's theorem.

(10 points) **3.** What is the coefficient of z^3 in the power series expansion of $f(z) = \sin z \cos z$ about the origin?

(10 points) **4.** What is the residue of

$$f(z) = \frac{(z-1) \cdot z}{\sin^2 z}$$

for the pole at the origin?

(10 points) **5.** What kind of singularity does the function $f(z) = e^z$ have at infinity?

(10 points) **6.** State the maximum principle.

(10 points) **7.** What are all the conformal self-maps of the unit disc?

(10 points) **8.** How many zeros does the function $f(z) = z^4 + 5z + 1$ have in the unit disc?

(10 points) **9.** What does the Schwarz-Pick lemma say?

(10 points) **10.** Give the first four terms of the Laurent expansion of

$$f(z) = \frac{\cos z \sin z}{z^2}$$

about the origin.