

PRACTICE THIRD MIDTERM

- (10 points) **1.** State the Schwarz lemma. Do not forget the uniqueness part.
- (10 points) **2.** Describe all the conformal maps from \mathbb{C} to \mathbb{C} .
- (10 points) **3.** Give an explicit conformal map of the unit disc $D(0, 1)$ to the upper halfplane $U = \{z \in \mathbb{C} : \text{Im } z > 0\}$.
- (10 points) **4.** Describe all the conformal self-maps of the annulus $A = \{z \in \mathbb{C} : 1/2 < |z| < 2\}$.
- (10 points) **5.** Verify that $u(x, y) = e^x \cos y$ is harmonic. Find a real-valued, harmonic function $v(x, y)$ so that $u + iv$ is holomorphic.
- (10 points) **6.** Explain why a holomorphic function satisfies the maximum modulus principle.
- (10 points) **7.** What is the Poisson integral formula? State it explicitly.
- (10 points) **8.** State the boundary modulus minimum principle.
- (10 points) **9.** What is the Cayley transform? Write it explicitly.
- (10 points) **10.** State the mean value property for holomorphic functions.