Math 318  Spring 2015
— Assignments are due at the beginning of the class on the due date.
— No late assignments will be accepted.
— Please use letter sized paper, and write your name and student number on the front page.
— Don’t forget to staple your homework!

Assignment 10  Due date: April 17, 2015

All numbered problems are from Shifrin.

**Section 5.3**  2, 5

**Section 5.3**  6(a) continued...

For the matrix  
\[
A = \begin{pmatrix}
1 & 3 \\
3 & 13
\end{pmatrix}
\]

you have computed the eigenvalues in the previous assignment. Now verify that the matrix may indeed be orthogonally diagonalized. That is, there exist orthogonal matrix  \( P \) and diagonal matrix  \( D \) such that

\[
A = PDP^{-1}.
\]

**Section 5.4**  1(a) (b)

Think about part (c), but you do have to hand in an answer.

**Section 5.4**  18

Note: Instead of using the constraint  \( ||x|| = 1 \), it is easier to use  \( ||x||^2 = 1 \).

**Section 5.4**  21, 28

**Problem-not-from-the-text.** Consider the point  \( a = (0,0,1)^T \) in \( \mathbb{R}^3 \). Define the function

\[
f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x) = ||x - a||^2
\]

(a) Compute the Hessian matrix of  \( f \).

(b) Restrict  \( f \) to the xy-plane, show that the origin  \( 0 \) is a constrained critical point.

(c) Show that the 2x2 topleft block of  \( \text{Hess}(f)(0) \) is positive-definite. This is called the constrained Hessian matrix of  \( f \).

Note: We conclude that  \( 0 \) is a local minimum of  \( f \) constrained to the xy-plane. That is, on the xy-plane,  \( 0 \) is the closest point  \( a \), which is obvious.