2016 Spring Math 430 Assignment 1. Due: Feb 1, 2016

Student No.

Name:

All problems, unless otherwise specified, are from A First Course in Abstract Algebra 7ed by Fraleigh. You do NOT need to hand in solutions to the problems in parentheses.

Extra problem.

1. Let $(V, +, \cdot)$ be a vector space over a field $\mathbb{F}$, and let W be a subspace of V. We define an equivalence relation $\sim$ on V: for $v_1, v_2 \in V$, we say $v_1 \sim v_2$ if $v_1 - v_2 \in W$. The quotient vector space V/W, as a set, is defined to be the quotient of this equivalence relation.

a. Show that $\sim$ is indeed an equivalence relation.

Now we give a vector space structure on V/W.

- $[0] \in V/W$ is the zero vector.
- For $[v_1], [v_2] \in V/W$, we define $[v_1] + [v_2] = [v_1 + v_2]$.
- For $[v] \in V/W$ and $\lambda \in \mathbb{F}$, we define $\lambda \cdot [v] = [\lambda \cdot v]$.

b. Show that V/W is indeed a vector space.

c. Now assume that the dimension of V is finite. Show that

$$\dim(V/W) = \dim V - \dim W.$$

§4 (16), (18), (25), 29, 32, 41

§5 22, (23), (25), (39), 43, 51, 52, 53, (57)

Note: For 52, the center of a group G is usually written as $Z(G)$.