2016 Spring Math 430   Assignment 5.   Due: N/A

All problems, unless otherwise specified, are from A First Course in Abstract Algebra 7ed by Fraleigh. For this assignment, you do NOT need to hand in the solutions, but the materials covered will be on Test 1 on Feb 29.

Extra problems.

1. Let S be a subset of a group G. The centralizer of S in G is defined to be

$$C_G(S) = \{g \in G \mid sg = gs, \forall s \in S\};$$

and the normalizer of S in G is defined to be

$$N_G(S) = \{g \in G \mid Sg = gS\}.$$

a) Show that $C_G(S)$ and $N_G(S)$ are subgroups of G.
b) Show that if H is a subgroup of G, then $H \triangleleft N_G(S)$ and $N_G(S)$ is the largest such group, that is, if $H \triangleleft G' < G$, then $G' < N_G(S)$.
c) Show that if H is a subgroup of G, then $C_G(H)$ is a normal subgroup of $N_G(H)$. What happens if H is not a subgroup of G?

§13   (10), (12), (13), (17), (19), (21), (32), (44), (45), (47), (49), (50), (52), (53)

§14   (6), (7), (23), (27), (28), (31), (37), (39)

Remark. The result of 37b states that for a group G, the group of inner automoprhisms $\mathrm{Inn}(G)$ is a normal subgroup of the automorphism group $\mathrm{Aut}(G)$. The group of outer automorphisms of G is defined to be

$$\mathrm{Out}(G) = \mathrm{Aut}(G)/\mathrm{Inn}(G).$$

Here is a challenge question. What is $\mathrm{Out}(S_3)$?