2016 Spring Math 430 Assignment 6. Due: March 9, 2016

Student No.

Name:

All problems, unless otherwise specified, are from A First Course in Abstract Algebra 7ed by Fraleigh. You do NOT need to hand in solutions to the problems in parentheses, but you need to hand in solution to the extra problems if there is any.

Extra problems.

1. **Definition.** Let H and N be groups, and let $\phi : H \to \mathrm{Aut}(N)$ be a group homomorphism, i.e. H acts on N. The semi-direct product of H and N, denoted by $H \ltimes_\phi N$, is defined as follows:

- as a set, $H \ltimes_\phi N = H \times N$;
- and the group operator is defined as

$$\begin{aligned} H \ltimes_\phi N \quad \times \quad H \ltimes_\phi N \quad &\longrightarrow \quad H \ltimes_\phi N, \\ (h_1, n_1) \cdot (h_2, n_2) &= (h_1 h_2, n_1 \phi_{h_1}(n_2)). \end{aligned}$$

a) Show that $H \ltimes_\phi N$ is indeed a group.

b) Show that N, or more precisely $\{e_H\} \times N$, is a normal subgroup of $H \ltimes_\phi N$.

c) Find the group homomorphism $\phi : \mathbb{Z}/2\mathbb{Z} \to \mathrm{Aut}(\mathbb{Z}/n\mathbb{Z})$ such that

$$D_n \cong \mathbb{Z}/2\mathbb{Z} \ltimes_\phi \mathbb{Z}/n\mathbb{Z}.$$

§15 (8), (10), (13), (19), 35, 36, 37, (38), (39), (40), 42

Note: The method outlined in 15.39 to prove that A_n is simple for $n \geq 5$ can be found in other textbooks, e.g. p.149 of Dummit-Foote 3 ed. I encourage you to at least read the proof.