

Student No.

Name:

All problems, unless otherwise specified, are from A First Course in Abstract Algebra 7ed by Fraleigh. You do NOT need to hand in solutions to the problems in parentheses, but you need to hand in solution to the extra problems if there is any.

Extra problems.

1. Recall that a group G is called nilpotent if its ascending central series is finite and terminates at G . The Heisenberg group H is defined as follows:

$$H = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}.$$

a. Show that the center Z of H is

$$Z = \left\{ \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mid b \in \mathbb{R} \right\}.$$

b. Show that for $g_1, g_2 \in H$, $g_1 g_2 g_1^{-1} g_2^{-1} \in Z$.

c. Conclude that H is nilpotent.

§35 (3), (5), (12), 14, (17), (22), 23, (24), 25, (26), 28, 29