

Math 132: Calculus II
Midterm Test 2

ANSWERS PROVIDED
at the end

Wednesday, 4 March 2015

15 Problems on 1 + 15 + 1 Pages

2 hours

You may not use any reference materials or electronic devices.

1. Find the average value of the function $f(x) = \sin(x)$ on the interval $0 \leq x \leq \pi$.

(a) 0

(b) $1/2$

(c) 1

(d) $1/\pi$

(e) $2/\pi$

(f) $\pi/2$

2. Find the volume of the solid formed by rotating the plane region **above** the curve $y = |x|$ and **below** the curve $y = \sqrt{2 - x^2}$ about the x axis.

(a) 2π

(b) 4π

(c) 8π

(d) $2\pi/3$

(e) $4\pi/3$

(f) $8\pi/3$

3. Find the volume of the solid formed by rotating the plane region **above** the curve $y = x$ and **below** the curve $y = 3x - x^2$ about the y axis.

- (a) 4π
- (b) 8π
- (c) 16π
- (d) $4\pi/3$
- (e) $8\pi/3$
- (f) $16\pi/3$

4. Find the centroid of the triangle with vertices $(0, 0)$, $(4, 0)$, and $(4, 3)$.

(a) $(2/3, 1/3)$

(b) $(4/3, 1/2)$

(c) $(8/3, 1)$

(d) $(8/3, 3/2)$

(e) $(4, 3/2)$

(f) $(4, 2/3)$

5. Find the y coordinate (\bar{y}) of the centroid of the plane region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and the curve $x^2 + y^2 = 1$ where $x \geq 0$ and $y \geq 0$. (Hint: it is a quarter circle.)

- (a) $\frac{3}{4\pi}$
- (b) $\frac{4\pi}{3}$
- (c) $\frac{3\pi}{4}$
- (d) $\frac{4}{3\pi}$
- (e) $\frac{1}{3}$
- (f) $\frac{\pi}{4}$

6. How much work is done filling a spherical water tank of radius 3m, whose center is 10m above ground, if gravity is approximated as a constant 10 N/kg and the water, which has mass 1000 kg/m^3 , starts at ground level?

(a) $1,800,000\pi \text{ J}$

(b) $2,700,000\pi \text{ J}$

(c) $3,600,000\pi \text{ J}$

(d) $9,000,000\pi \text{ J}$

(e) $18,000,000\pi \text{ J}$

(f) $27,000,000\pi \text{ J}$

7. A spring whose natural length is 10cm takes 16 J of work to stretch to 14cm. How much work does it take to stretch the spring from 15cm to 20cm?

- (a) 25 J
- (b) 50 J
- (c) 75 J
- (d) 100 J
- (e) 150 J
- (f) 200 J

8. On a dense planet with radius 1000km, the force of gravity at the surface is 3 N/kg. How much energy is required to lift 1 kg of mass from the surface to infinity? Assume that gravitation outside a planet obeys the inverse square law $F = kM/r^2$, where k is some constant, M is the mass of the object, and r is the distance between the object and the center of the planet.
- (a) 15,000,000 J
 - (b) 12,000,000 J
 - (c) 9,000,000 J
 - (d) 6,000,000 J
 - (e) 3,000,000 J
 - (f) It will require infinite work.

9. What is the length of the plane curve $y = \frac{2}{3}x^{3/2}$, $0 \leq x \leq 15$?

(a) $128/3$

(b) $52/3$

(c) 19

(d) 42

(e) 52

(f) 128

10. What is the length of a cable described by the equation $y = 5 + \cosh x$, $-2 \leq x \leq 2$?

(a) $5 \sinh 2$

(b) $5 \cosh 2$

(c) $e - e^{-1}$

(d) $e + e^{-1}$

(e) $e^2 - e^{-2}$

(f) $e^2 + e^{-2}$

11. Find the area of the surface obtained by rotating the curve $y = \frac{1}{3}x^3$, $0 \leq x \leq 3^{1/4}$, about the x axis.

(a) 7π

(b) 9π

(c) $7\pi/9$

(d) $9\pi/7$

(e) $\pi/9$

(f) $\pi/7$

12. Find the solution to the following initial value problem:

$$\frac{dy}{dx} = 7y, \quad y(2) = 12$$

(a) $y(x) = 12e^{7(x+2)}$

(b) $y(x) = 12e^{7(x-2)}$

(c) $y(x) = 12 \times 7^{x+2}$

(d) $y(x) = 12 \times 7^{x-2}$

(e) $y(x) = 11 + e^{7(x+2)}$

(f) $y(x) = 11 + e^{7(x-2)}$

13. Compute

$$\int_1^e x^{-1} \log x \, dx$$

- (a) $1/e$
- (b) $e/2$
- (c) e
- (d) $1/e^2$
- (e) $1/2$
- (f) 1

14. Use integration by parts to compute

$$\int_1^e x \log x \, dx$$

- (a) $(1 + e^2)/2$
- (b) $(1 + e^2)/4$
- (c) $(e^2 - 1)/2$
- (d) $(e^2 - 1)/4$
- (e) $e^2/2$
- (f) $e^2 + e + 1$

15. Use integration by parts to compute

$$\int_0^{\pi/2} x^2 \sin x \, dx$$

- (a) $\frac{\pi}{2} + 1$
- (b) $\frac{\pi}{2} - 1$
- (c) $2 - \frac{\pi}{2}$
- (d) $2 + \frac{\pi}{2}$
- (e) $\pi/2$
- (f) 1

Correct answers:

[1] "e"

[2] "f"

[3] "e"

[4] "c"

[5] "d"

[6] "c"

[7] "c"

[8] "e"

[9] "d"

[10] "e"

[11] "c"

[12] "b"

[13] "e"

[14] "b"

[15] no correct answer