Math 132: Calculus II
Midterm Test 3

ANSWERS PROVIDED
at the end

Wednesday, 8 April 2015

15 Problems on 1 + 15 + 1 Pages

2 hours

You may not use any reference materials or electronic devices.

1. Evaluate the indefinite integral

$$\int \cos(12x)\cos(7x)\,dx$$

(a) $\frac{1}{38}\sin(19x) + \frac{1}{10}\sin(5x) + C$

(b) $\frac{1}{19}\sin(19x) + \frac{1}{5}\sin(5x) + C$

(c) $\frac{1}{2}\sin(12x) + \frac{1}{2}\sin(7x) + C$

(d) $\frac{1}{38}\cos(19x) + \frac{1}{10}\cos(5x) + C$

(e) $\frac{1}{19}\cos(19x) + \frac{1}{5}\cos(5x) + C$

(f) $\frac{1}{2}\cos(19x) + \frac{1}{2}\sin(5x) + C$

2. Find the antiderivative

$$\int \frac{dx}{x^2\sqrt{1-x^2}}$$

(a) $-x\sin^{-1}(x) + C$

(b) $x\cos^{-1}(x) + C$

(c) $+\frac{x}{\sqrt{1-x^2}} + C$

(d) $-\frac{x}{\sqrt{1-x^2}} + C$

(e) $+\frac{\sqrt{1-x^2}}{x} + C$

(f) $-\frac{\sqrt{1-x^2}}{x} + C$

3. Find the antiderivative

$$\int \frac{x^3 - 1}{x^2 + 3x + 2}\, dx$$

(a) $\frac{x^2}{2} + 3x + 2\log|x+1| + 9\log|x+2| + C$

(b) $\frac{x^2}{2} - 3x - 2\log|x+1| + 9\log|x+2| + C$

(c) $\frac{x^2}{2x+3} + C$

(d) $\frac{3x^2}{2x+3} + C$

(e) $\frac{1}{4}x^4 - x + \log|x^2 + 3x + 2| + C$

(f) $\frac{1}{4}x^4 - x + \log|2x + 3| + C$

4. Suppose that a function f has a continuous second derivative that satisfies $|f''(x)| \leq 12$ on the interval $0 \leq x \leq 10$. Find the smallest value of n such that the trapezoid rule approximation to $\int_0^{10} f(x)\,dx$, using n subintervals, will have error less than 10^{-7}.

(a) 300

(b) 500

(c) 700

(d) 10^3

(e) 10^5

(f) 10^7

5. Suppose that a function f takes the following values:

x	0	1	2	3	4
$f(x)$	1	0	1	4	9

Use Simpson's rule with as many subintervals as possible to approximate $\int_0^4 f(x)\,dx$.

(a) 7/6

(b) 7/3

(c) 14/3

(d) 16/3

(e) 26/3

(f) 28/3

6. Evaluate the improper Riemann integral

$$\int_0^\infty e^{-0.001x}\,dx$$

(a) $1000/e$

(b) 1000

(c) $1000e$

(d) $1/(1-0.001e)$

(e) $1/(1+0.001e)$

(f) It does not converge.

7. Evaluate the improper Riemann integral

$$\int_0^1 x^{-0.001e}\,dx$$

(a) $1/(1-0.001e)$

(b) $1/(1+0.001e)$

(c) $1000/e$

(d) 1000

(e) $1000e$

(f) It does not converge.

8. Evaluate the improper Riemann integral

$$\int_0^\infty xe^{-x^2}\,dx$$

(a) $1/2$

(b) 1

(c) $\sqrt{\pi}$

(d) $2\sqrt{\pi}$

(e) $\sqrt{\pi}/2$

(f) It does not converge.

9. Find the unique convergent sequence $\{a_n : n = 1, 2, 3, \ldots\}$, given a_n:

(a) $a_n = \frac{n!}{1000^n}$

(b) $a_n = \frac{e^{n!}}{n!}$

(c) $a_n = \frac{n!}{n^2}$

(d) $a_n = \frac{e^n}{2^n}$

(e) $a_n = \frac{(n+2)(n+3)}{1+n^2}$

(f) $a_n = \frac{(n+2)(n+3)}{1000+n}$

10. Find the unique divergent sequence $\{a_n : n = 1, 2, 3, \ldots\}$, given a_n:

(a) $a_n = \frac{1}{n} - \frac{1}{n+1}$

(b) $a_n = \log(n+1) - \log(n)$

(c) $a_n = \sqrt{n+1} - \sqrt{n}$

(d) $a_n = \tan(\pi n)$

(e) $a_n = \sin(\pi n)$

(f) $a_n = \cos(\pi n)$

11. Find the limit of the sequence $\{a_n : n = 1, 2, 3, \ldots\}$, where

$$a_n = n \tan(2/n)$$

(a) 0

(b) 1/2

(c) 1

(d) 2

(e) The sequence diverges to infinity.

(f) The sequence diverges, but not to infinity.

12. Evaluate the infinite series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

(a) 10/9

(b) 11/18

(c) 1

(d) 1/2

(e) 1/3

(f) The sum diverges to positive infinity.

13. Find the value of the infinite series

$$\sum_{n=0}^{\infty} \frac{1^n + 2^n + 3^n + 4^n}{(1+2+3+4)^n}$$

(a) $\frac{1}{10}(\frac{1}{9} + \frac{1}{8} + \frac{1}{7} + \frac{1}{6})$

(b) $5(\frac{1}{9} + \frac{1}{8} + \frac{1}{7} + \frac{1}{6})$

(c) $10(\frac{1}{9} + \frac{1}{8} + \frac{1}{7} + \frac{1}{6})$

(d) $\frac{1}{5}(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4})$

(e) $5(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4})$

(f) $10(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4})$

14. Find the unique convergent series among the following:

(a) $\sum_{n=1}^{\infty} 2^{1/n}$

(b) $\sum_{n=1}^{\infty}(-1)^n 2^{1/n}$

(c) $\sum_{n=1}^{\infty}(-1)^n 2^n$

(d) $\sum_{n=1}^{\infty}(-1)^n/n$

(e) $\sum_{n=1}^{\infty}(-n)^n$

(f) $\sum_{n=1}^{\infty} \frac{1}{n}\log(n)$

15. Find the unique divergent series among the following:

(a) $\sum_{n=1}^{\infty} n^{-n}$

(b) $\sum_{n=1}^{\infty}(n + \log n)^{-2}$

(c) $\sum_{n=1}^{\infty} ne^{-n}$

(d) $\sum_{n=1}^{\infty}(-1)^n \tan(1/n)$

(e) $\sum_{n=1}^{\infty}(-1)^n \sin(1/n)$

(f) $\sum_{n=1}^{\infty}(-1)^n \cos(1/n)$

Correct answers:

[1] “a”

[2] “f”

[3] “b”

[4] “e”

[5] “f”

[6] “b”

[7] “a”

[8] “a”

[9] “e”

[10] “f”

[11] “d”

[12] “b”

[13] “c”

[14] “d”

[15] “f”