QUIZ 1

Show your work not just your final answer

(1) Solve the system of linear equations.
\[
\begin{align*}
  x + y &= 6 \\
  5x - 4y &= 39 \\
  16x - 11y &= 123
\end{align*}
\]

(2) Consider a linear system whose augmented matrix is,
\[
\begin{bmatrix}
1 & 1 & 3 & 3 \\
1 & 2 & -5 & 1 \\
6 & 15 & k & 1
\end{bmatrix}
\]
Find \( k \) so that the system is inconsistent.

(3) How many pivots does the following matrix have?
\[
\begin{bmatrix}
0 & 1 & 0 & -8 \\
0 & 0 & 1 & 6
\end{bmatrix}
\]

(4) Decide whether the augmented matrix given in echelon form below is consistent (that is, the corresponding system of linear equations is consistent) and if so, does it have a unique solution.
\[
\begin{bmatrix}
1 & 0 & 0 & -4 \\
0 & 1 & 0 & -3 \\
0 & 0 & 0 & -2
\end{bmatrix}
\]

(5) Let \( \mathbf{x}, \mathbf{y} \) be vectors and let \( \mathbf{z} = p\mathbf{x} + q\mathbf{y} \), where \( p, q \) are real numbers. For what values of \( p, q \) must \( \text{Span}\{\mathbf{x}, \mathbf{z}\} \) be equal to \( \text{Span}\{\mathbf{y}, \mathbf{z}\} \)?

(6) Let \( A = \begin{bmatrix} -8 & 3 \\ -8 & -1 \end{bmatrix} \), \( \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \) and \( \mathbf{b} = \begin{bmatrix} -39 \\ -19 \end{bmatrix} \). Solve for \( \mathbf{x} \), given \( A\mathbf{x} = \mathbf{b} \).

(7) Solve the equation \(-8x - 2y + 9z = 0\) in parametric form. In other words, find two solutions \( \mathbf{u}, \mathbf{v} \), 3-vectors, so that all solutions are of the form \( s\mathbf{u} + t\mathbf{v} \) for some constants \( s, t \).
(8) Consider the following system of linear equations.

\[
\begin{align*}
2w - x + y &- z = 4 \\
2w + x - y + z & = 4 \\
3w & = 8 \\
3w - 3x + 3y - 3z & = 4
\end{align*}
\]

Write solutions to these as \( w, x \) in terms of \( y, z \).

(9) Let \( u = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \ v = \begin{bmatrix} -1 \\ 6 \\ 3 \end{bmatrix}, \ w = \begin{bmatrix} 2 \\ -11 \\ 6 \end{bmatrix} \). Decide whether these are linearly independent.

(10) Let

\[
A = \begin{bmatrix}
-1 & -3 & 1 \\
1 & 2 & 0 \\
-1 & 2 & -4
\end{bmatrix}
\]

Decide whether \( Ax = 0 \) has a non-trivial (non-zero) solution.

(11) Let \( A = \begin{bmatrix} -4 & -4 & 5 \\ 5 & -6 & -4 \\ 6 & -5 & -4 \end{bmatrix} \) and \( b = \begin{bmatrix} 21 \\ -2 \\ -11 \end{bmatrix} \). Define a linear transformation \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) by \( T(x) = Ax \). Find a vector \( x \) such that \( T(x) = b \).

(12) Let \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) be the linear transformation, \( T(x) = \begin{bmatrix} x_1 - x_2 \\ x_2 - x_3 \\ x_3 - x_1 \end{bmatrix} \).

Is this onto?