Ma 322: Biostatistics
Homework Assignment 1
Prof. Wickerhauser

Begin by obtaining access to the R software package, either by downloading a copy onto your computer or else by finding a computer with a working installation. Any version number 2.2 or greater should work. You may then also download R Studio for its convenience.

Read Chapter 6, pages 60–79, of our e-text to review basic principles of probability. Consult Chapters 1-5 as needed to find function names and syntax to solve the computation problems below.

1. Let \texttt{SEED} be your student ID number. Generate 100 samples from a standard normal density using the following \texttt{R} commands:

\begin{verbatim}
set.seed(SEED); x <- rnorm(100)
\end{verbatim}

(a) Plot the histogram of \(x\) using \texttt{R} defaults.

(b) Plot the histogram of \(x\) using enough integer breakpoints to include all samples, with bins that include their left endpoint but not their right endpoint. HINT: this is not the default for \texttt{hist()} in \texttt{R}. It will be necessary to check \texttt{range(x)}, and to set both the \texttt{right} and \texttt{include.lowest} parameters to non-default values. Read the documentation page!

(c) Calculate the mean and median of the samples.

(d) Calculate the standard deviation, variance, and mean absolute deviation of the samples.

2. Reuse the population \(x\) generated in Problem 1, keeping the original ordering. Set the random number generator seed to your student ID before each sampling (that is, before each \texttt{rnorm()} and each \texttt{sample()}) to get reproducible results.

(a) Pick a random subsample of 10 values, without replacement, and compute the sample mean and sample median for those 10 values.

(b) Pick a random subsample of 10 values with replacement, then compute the sample mean and sample median for those 10 values.

3. Consider the following table of tree species in a complete count from a section of forest:
<table>
<thead>
<tr>
<th>Species</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>White Oak</td>
<td>40</td>
</tr>
<tr>
<td>Red Oak</td>
<td>33</td>
</tr>
<tr>
<td>Shagbark hickory</td>
<td>17</td>
</tr>
<tr>
<td>Black walnut</td>
<td>12</td>
</tr>
<tr>
<td>Basswood</td>
<td>13</td>
</tr>
<tr>
<td>Slippery Elm</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Use the Shannon index to express the tree species diversity. Compute the maximum Shannon diversity possible for this number of species, and then calculate the Shannon evenness for this table.

(b) Compute the Brillouin diversity index for the frequency table in the previous problem. Find the maximum Brillouin diversity, then calculate the Brillouin evenness.

4. A debate team has 8 Klingons and 7 Vulcans.

(a) How many distinct mixed pairs (one Klingon and one Vulcan) can be formed using members of the team?

(b) How many distinct practice matchups of two mixed pairs can be formed using members of the team?

5. A DNA modeling kit contains 13 base units: 4 A’s, 2 C’s, 4 G’s, and 3 T’s.

(a) How many distinct sequences of length 2 can be formed from this kit?

(b) How many distinct sequences of length 13 can be formed from this kit?

(c) How many distinct sequences of length 3 can be formed from this kit?

(d) How many distinct sequences of length 6 can be formed from this kit? Of length 9? Of length 12? (Hint: use deduct().)

6. Subsets $A, B, C, D, E$ satisfy $B \subset A$, $C \subset B$, $D \subset B$, $C \cap D = \emptyset$, and $A \cap E = \emptyset$.

(a) Depict the sets using a Venn diagram.

(b) Is $C \cap E = \emptyset$?

7. A standard set of 52 playing cards is divided into 4 suits of 13 ranks each: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king. The suits are called clubs, diamonds, hearts and spades, with clubs and spades being “black” and hearts and diamonds being “red.”

(a) Taking 1 card at random, what is the probability of drawing a king of spades? a black king? a face card (jack, queen, or king)?

(b) Taking 2 cards at random without replacement, what is the probability of drawing a pair of kings? a pair of clubs? a pair of black cards? a pair of cards of different ranks and suits?

(c) Taking 5 cards at random without replacement, what is the probability of drawing a “full house,” namely 3 cards of one rank and 2 cards of a second rank?