Ma 322: Biostatistics
Homework Assignment 3

Prof. Wickerhauser

Due Monday, February 15th, 2021

Read Chapter 8, pages 108–134 of our text.

1. Every morning, Rosencrantz flips a coin. If it turns up heads, he rises out of bed and rolls two dice to decide what he will have for breakfast. If the sum of the dice is more than 8 he has eggs, otherwise he has cereal.

   If the coin turns up tails, Rosencrantz sleeps for another hour and then has eggs for breakfast.

   (a) Assuming a fair coin and fair dice, what is the conditional probability that Rosencrantz eats cereal for breakfast given that the coin flip turned up heads?

   (b) Assuming a fair coin and fair dice, what is the probability that Rosencrantz eats eggs for breakfast? What is the probability that he eats cereal for breakfast?

2. Use the joint and marginal probability table 8-2 on page 114 of our text to answer the following questions:

   (a) What is P(G at P1)?

   (b) What is P(G at P2)?

   (c) What is P(G at both P1 and P2)?

   (d) What is P(T at P1 | G at P2)?

3. Suppose that X, Y are continuous random variables, each taking values in $[0, 1]$, with joint probability density function
$$f(x, y) = c(1 - xy),$$
   where c is a constant.

   (a) Find c.

   (b) Find the marginal density function $f_X(x)$.

   (c) Find the marginal density function $f_Y(y)$.

   (d) Are X and Y independent? Supply a proof or a counterexample to justify your answer.

   (e) Compute $P(x < \frac{1}{2} \mid y = \frac{1}{2})$.

4. Suppose that genotypes AA, Aa, and aa have respective occurence probabilities $P_{AA} = 0.1$, $P_{Aa} = 0.2$, and $P_{aa} = 0.7$ in a population of diploid organisms.

   (a) What is the probability of getting 1 AA, 2 Aa's, and 7 aa's in a random sample of 10 individuals from this population?

(b) What is the probability of getting no AA's in a random sample of 10 individuals from this population?

(c) Simulate taking 8 independent random samples of 10 individuals from this population and use the simulation to estimate P_{AA}, P_{Aa}, and P_{aa}.

5. (a) Let $B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$. Compute $\Sigma = B^T B$ and find Σ^{-1}.

   (b) Use `persp()` and `contour()` to display the bivariate normal density with mean $(0, 1)$ and covariance matrix Σ (from part a) over the range $[-9, 9] \times [-9, 9]$

   (c) Use `mvrnorm()` to simulate 1000 samples from the bivariate normal density of part b. Display the resulting scatterplot.

   (d) Display the histograms of the X and Y marginal densities of the simulation in part c.

   (e) Calculate the covariance matrix of the samples in part c. Hint: check your work by comparing the sample covariance matrix with Σ.