Ma 322: Biostatistics
Homework Assignment 5

Prof. Wickerhauser

Due Monday, March 1st, 2021

Read Chapter 10, "Stochastic Processes and Markov Chains," pages 160–184 of our text.

Note: Although our text has no index or table of contents, it is easy to locate words in the electronic version using the Find function of your favorite PDF reader.

1. Suppose that the list of variables $p = (p_1, \ldots, p_K)$ has a Dirichlet prior density

$$f_\alpha(p) \propto p_1^{\alpha_1 - 1} \cdots p_K^{\alpha_K - 1},$$

where $\alpha = (\alpha_1, \ldots, \alpha_K)$ is a list of shape parameters. We perform an experiment that yields counts $n = (n_1, \ldots, n_K)$ having the multinomial likelihood

$$L_n(p) \propto p_1^{n_1} \cdots p_K^{n_K}.$$

(a) For what values of α does one get a non-informative Dirichlet prior pdf?

(b) Determine the shape parameters for the posterior pdf $L_n(p)f_\alpha(p)$.

2. Suppose that 100 individuals selected randomly from a population are blood-typed and the results are 49 type O, 23 type A, 18 type B, and 10 type AB.

(a) Using a non-informative prior, and assuming Hardy-Weinberg equilibrium, generate a contour plot of the posterior pdf on the proportions p_A and p_B of the A and B blood-type alleles, respectively, in the population. HINT: see `r-eg-35.txt` on the class website.

(b) Find, at least approximately, the maximum-likelihood estimator of the proportion of A,B, and O alleles in the population.

3. Implement the function `Walk1d()` on p.167 of our text and graph three 100-step simulations starting from random seeds 123 and 4567 and 89012.

4. Implement the function `Walk2d()` on p.168 of our text and graph three 500-step simulations starting from random seeds 123 and 4567 and 89012.

5. A restless koala moves among three eucalyptus trees labeled 1, 2, and 3. A patient park ranger watches and makes notes every morning and evening on the koala's position, producing the following table:

Koala Tree-Change Counts

Morning Tree	Evening Tree	Count
1	1	3
1	2	12
1	3	8
2	1	10
2	2	5
2	3	20
3	1	13
3	2	4
3	3	7

(a) Treat the koala's movements as a Markov process and determine the transition matrix M from this table of counts.

(b) Starting with a uniform prior distribution on the three trees and assuming the koala's tree-change preferences remain the same, compute the posterior koala distribution, namely the stationary distribution determined by M.

6. Consider the following transition matrix for a 4-state Markov chain:

$$M = \begin{pmatrix} 0.2 & 0.1 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.1 & 0.2 \\ 0.4 & 0.3 & 0.2 & 0.1 \end{pmatrix}.$$

(a) Is M periodic or aperiodic?

(b) Is M irreducible?

(c) Is M ergodic?

(d) Does M have a stationary distribution?

(e) Is M reversible?

7. Consider the following transition matrix for a 5-state Markov chain:

$$F = \begin{pmatrix} 0.20 & 0.20 & 0.20 & 0.20 & 0.20 \\ 0.0 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0.0 & 0.0 & 0.33 & 0.33 & 0.34 \\ 0.0 & 0.0 & 0.0 & 0.50 & 0.50 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.00 \end{pmatrix}.$$

(a) Is F periodic or aperiodic?

(b) Is F irreducible?

(c) Is F ergodic?

(d) Does F have a stationary distribution?

(e) Is F reversible?