

Ma 322: Biostatistics

Solutions to Homework Assignment 2

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Due Monday, February 8th, 2021

Read Chapter 7, pages 80–107, of our e-text to review some basic probability density functions and their properties, concentrating especially on the normal pdf. Consult Chapters 1-5 as needed to find function names and syntax to solve the computation problems below.

1. On a single graph, plot the exponential pdf $p(t) = \lambda e^{-\lambda t}$ over the interval $0 \leq t \leq 3$ for the values $\lambda = 1.5$, $\lambda = 1$, and $\lambda = 0.5$.

Be sure to choose axes so that the maximum value of each pdf can be seen.

Solution: Use the following R commands:

```
t <- seq(0,3,by=0.01);
plot(t,dexp(t,rate=1.5), type='l', col='black');
lines(t,dexp(t,rate=1.0), col='red');
lines(t,dexp(t,rate=0.5), col='blue');
```

They produce the output below, stored in a file with `pdf()`. Using the largest rate $\lambda = 1.5$ first scales the plots properly. \square

2. On a single graph, plot the normal pdf $p(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-\mu)^2/2\sigma^2}$ over the interval $-3 \leq t \leq 3$ for $\mu = 0$ and the values $\sigma = 1.5$, $\sigma = 1$, and $\sigma = 0.5$.

Be sure to choose axes so that the maximum value of each pdf can be seen.

Solution: Use the following R commands:

```
t <- seq(-3,3,by=0.01);
plot(t,dnorm(t,sd=0.5), type='l', col='black');
lines(t,dnorm(t,sd=1.0), col='red');
lines(t,dnorm(t,sd=1.5), col='blue');
```

They produce the output below, stored in a file with `pdf()`. Using the smallest standard deviation $\sigma = 0.5$ first scales the plots properly. \square

3. On a single graph, plot the Gamma pdf $p(t) = \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} e^{-t/\beta}$ over the interval $0 \leq t \leq 10$ for the values $(\alpha, \beta) = (1, 1)$, $(\alpha, \beta) = (1, 2)$, $(\alpha, \beta) = (2, 1)$, and $(\alpha, \beta) = (2, 2)$.

Be sure to choose axes so that the maximum value of each pdf can be seen.

Solution: Use the following R commands:

```
t <- seq(0,10,by=0.01);
plot(t,dgamma(t,shape=1,scale=1), type='l', col='black');
lines(t,dgamma(t,shape=1,scale=2), col='red');
lines(t,dgamma(t,shape=2,scale=1), col='green');
lines(t,dgamma(t,shape=2,scale=2), col='blue');
```

They produce the output below, stored in a file with `pdf()`. Using the smallest shape and scale values $(\alpha, \beta) = (1, 1)$ first scales the plots properly. \square

4. On a single graph, plot the Beta pdf $p(t) = \frac{1}{B(\alpha, \beta)} t^{\alpha-1} (1-t)^{\beta-1}$ over the interval $0 \leq t \leq 1$ for the values $(\alpha, \beta) = (1, 1)$, $(\alpha, \beta) = (2, 5)$, $(\alpha, \beta) = (8, 2)$, and $(\alpha, \beta) = (8, 5)$.

Be sure to choose axes so that the maximum value of each pdf can be seen.

Solution: Use the following R commands:

```
t <- seq(0,1,by=0.01);
plot(t,dbeta(t,shape1=8,shape2=2), type='l', col='black');
lines(t,dbeta(t,shape1=2,shape2=5), col='red');
lines(t,dbeta(t,shape1=1,shape2=1), col='green');
lines(t,dbeta(t,shape1=8,shape2=5), col='blue');
```

They produce the output below, stored in a file with `pdf()`. Using the most lopsided shape and scale values $(\alpha, \beta) = (8, 2)$ first scales the plots properly. \square

5. On a single graph, plot the Chi squared (χ^2) pdf $p(t) = \frac{1}{2^{k/2}\Gamma(k/2)} t^{k/2-1} e^{-t/2}$ over the interval $0 \leq t \leq 10$ for the values $k = 2$, $k = 3$, and $k = 7$.

Be sure to choose axes so that the maximum value of each pdf can be seen.

Solution: Use the following R code:

```
x<-seq(0,10,by=0.1); plot(x,dchisq(x,df=2),type="l");
lines(x,dchisq(x,df=3));lines(x,dchisq(x,df=7));
```

They produce the output below, stored in a file with `pdf()`. Using the fewest degrees of freedom $k = 2$ first scales the plots properly. \square

6. On a single graph, plot the binomial pdf $p(k) = \binom{n}{k} s^k (1-s)^{n-k}$ for $n = 100$ Bernoulli trials over the interval $0 \leq k \leq n$ for the success rate values $s = 0.1$, $s = 0.2$, $s = 0.5$, and $s = 0.9$.

Be sure to choose axes so that the maximum value of each pdf can be seen.

Solution: Use the following R commands:

```
n<-100;
k <- seq(0,n,by=1);
plot(k,dbinom(k,size=n, prob=0.1), col='black');
points(k,dbinom(k,size=n, prob=0.2), col='red');
points(k,dbinom(k,size=n, prob=0.5), col='green');
points(k,dbinom(k,size=n, prob=0.9), col='blue');
```

They produce the output below, stored in a file with `pdf()`. Using the lowest success rate $s = 0.1$ first scales the plots properly. \square

7. On a single graph, plot the Poisson pdf $p(k) = e^{-\lambda}\lambda^k/k!$ over the interval $0 \leq k \leq 100$ for the mean count values $\lambda = 5$, $\lambda = 10$, $\lambda = 20$, and $\lambda = 50$.

Be sure to choose axes so that the maximum value of each pdf can be seen.

Solution: Use the following R commands:

```
k <- seq(0,100,by=1);
plot(k,dpois(k,lambda=5), col='black');
points(k,dpois(k,lambda=10), col='red');
points(k,dpois(k,lambda=20), col='green');
points(k,dpois(k,lambda=50), col='blue');
```

They produce the output below, stored in a file with `pdf()`. Using the lowest mean count $\lambda = 5$ first scales the plots properly. □

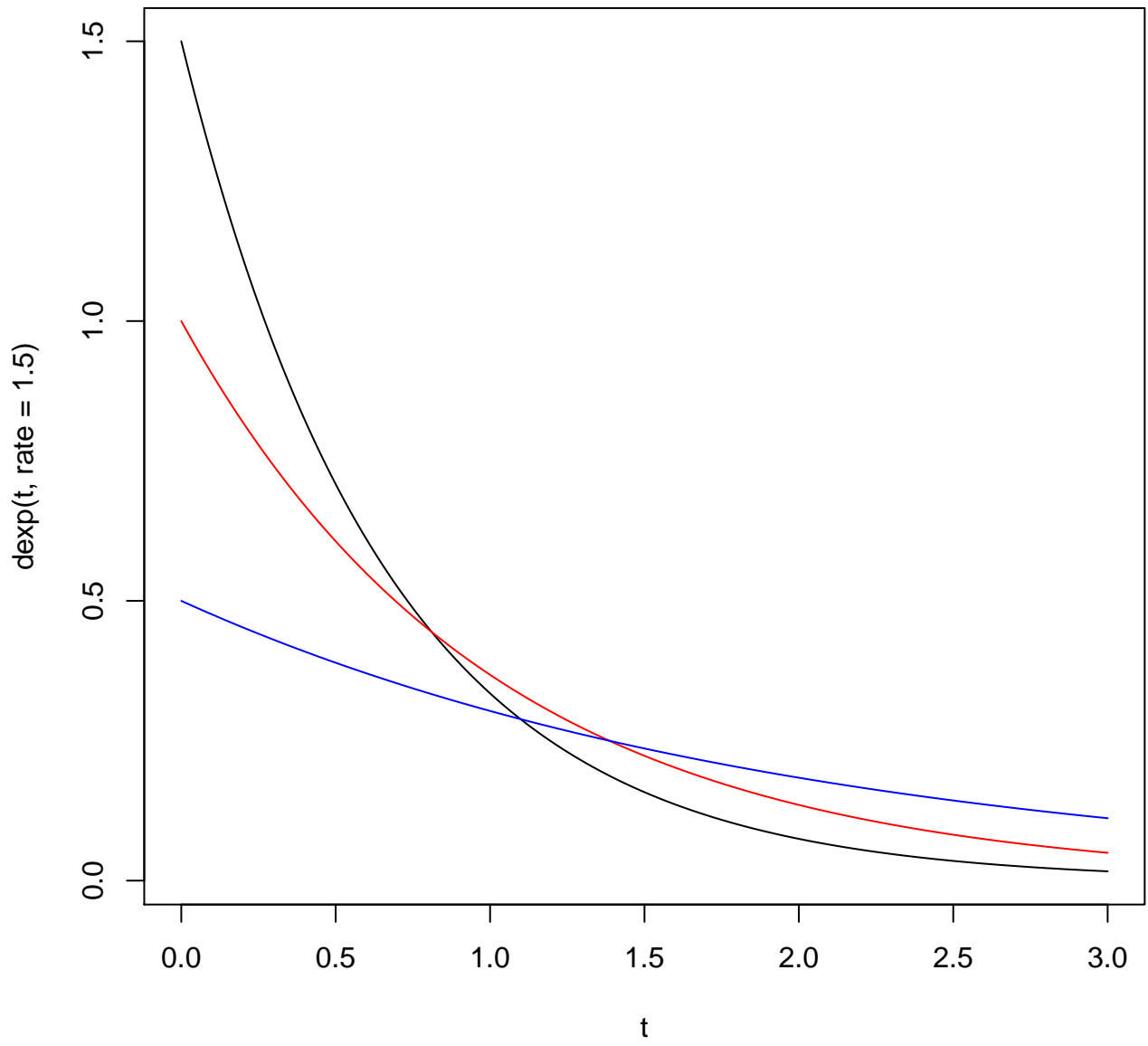


Figure 1: Graph for Problem 1

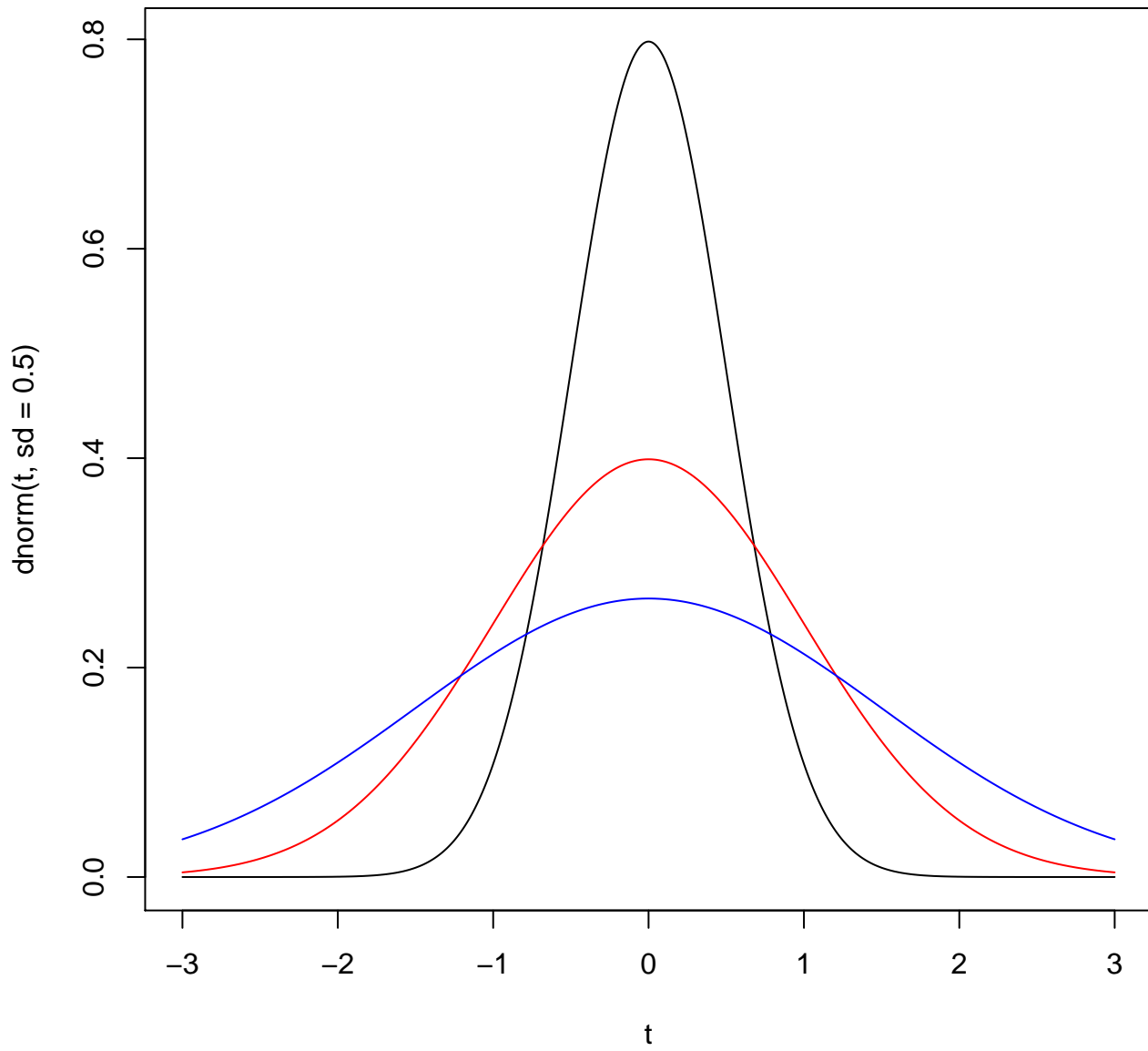


Figure 2: Graph for Problem 2

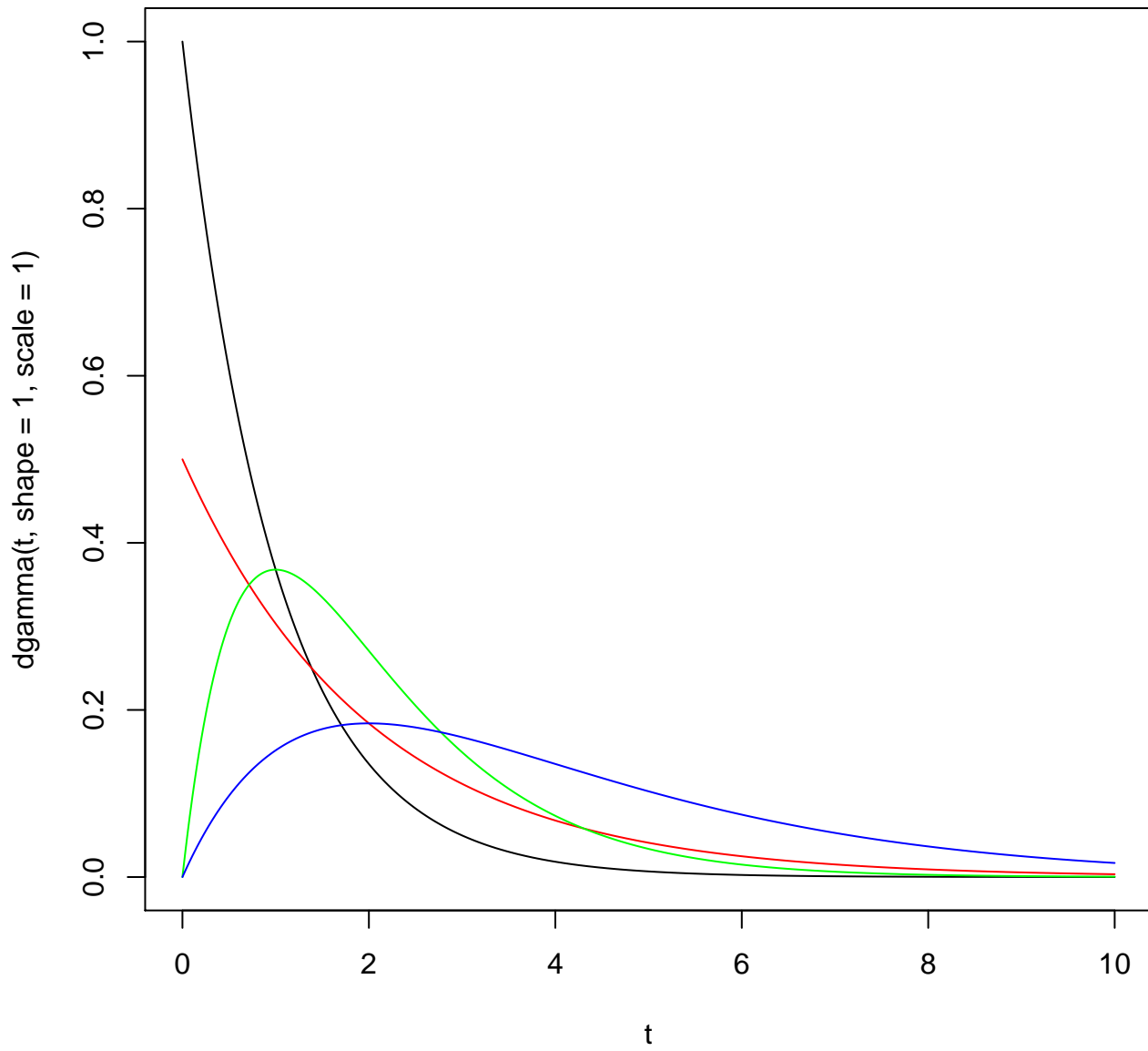


Figure 3: Graph for Problem 3

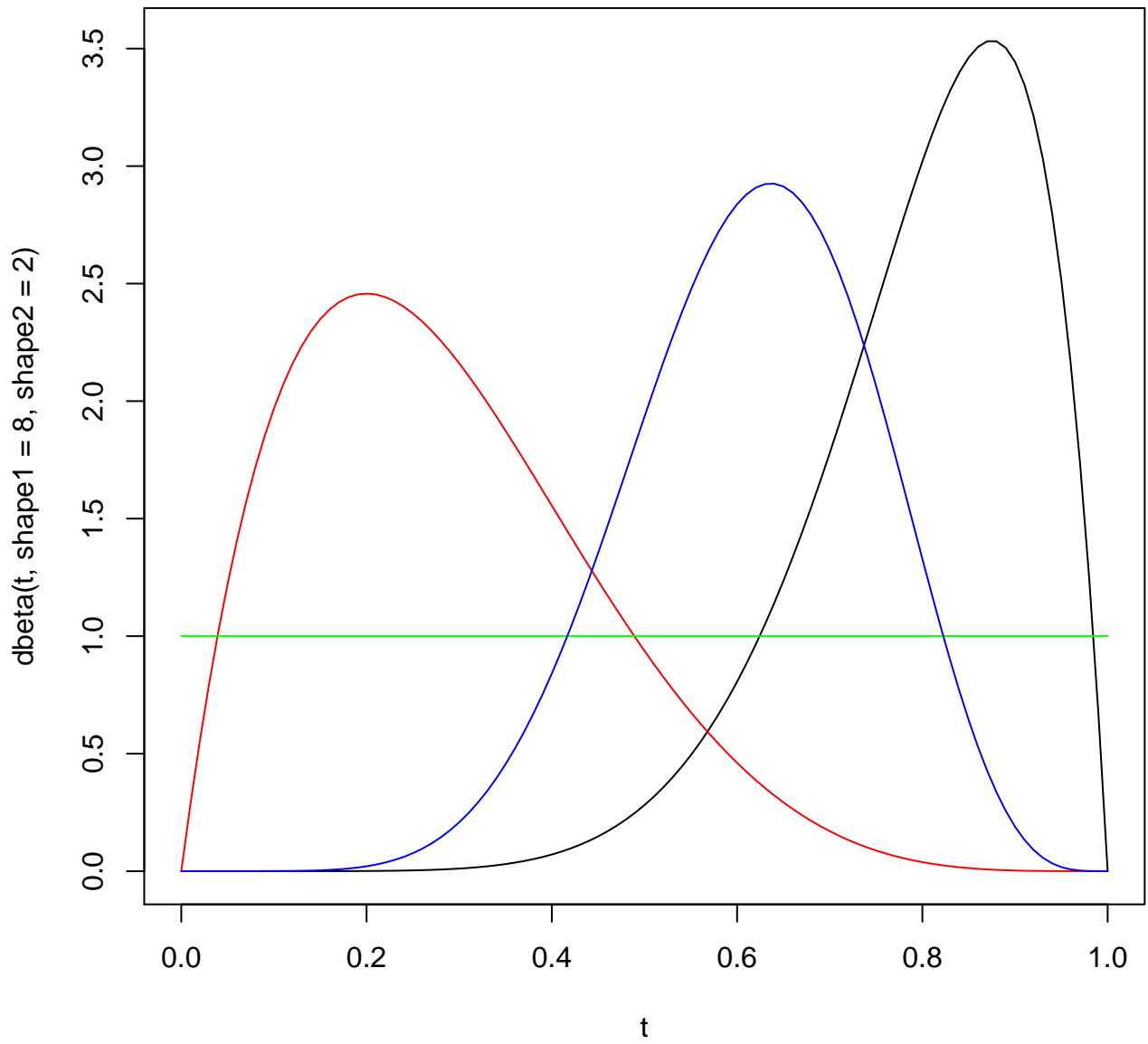


Figure 4: Graph for Problem 4

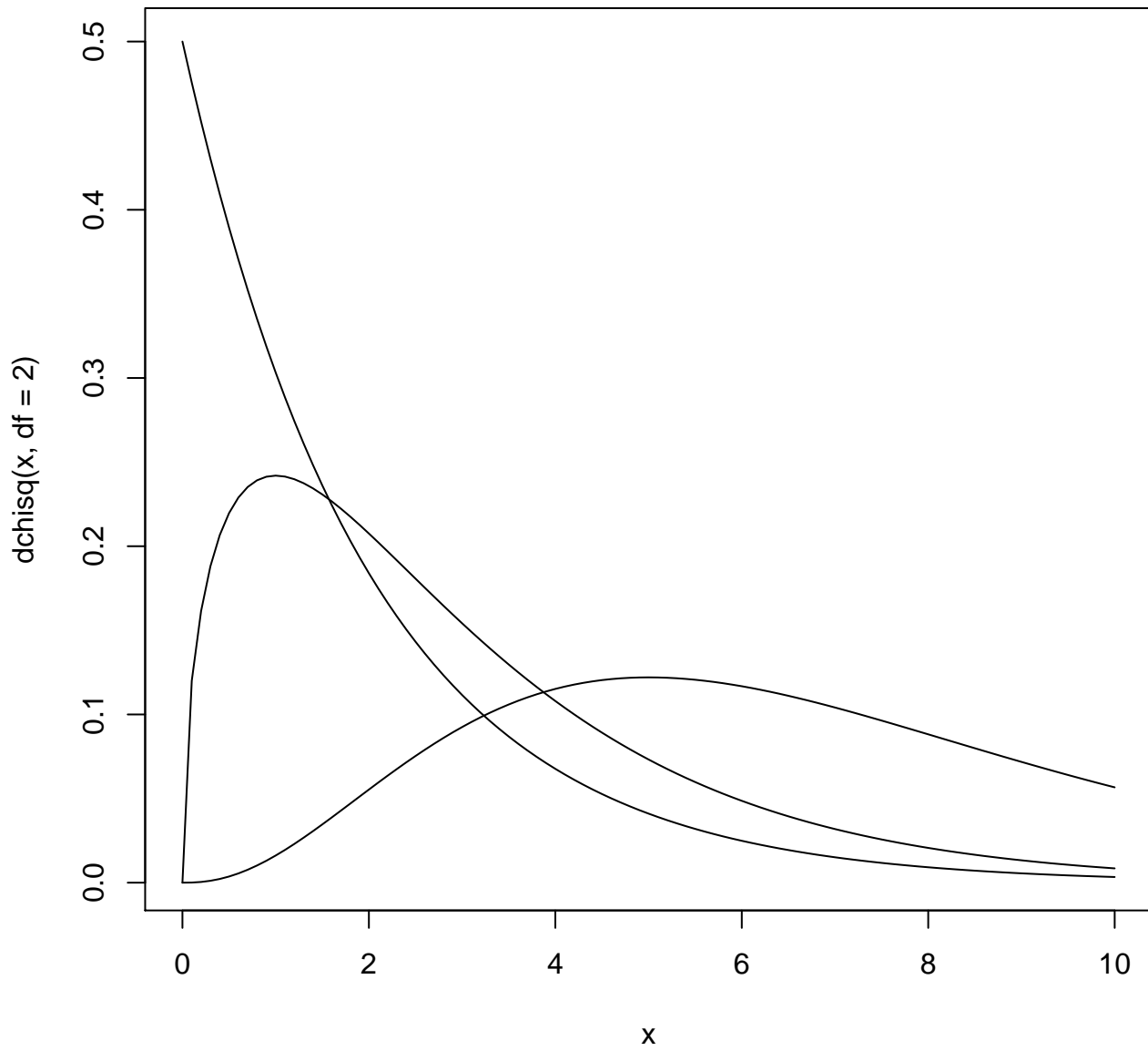


Figure 5: Graph for Problem 5

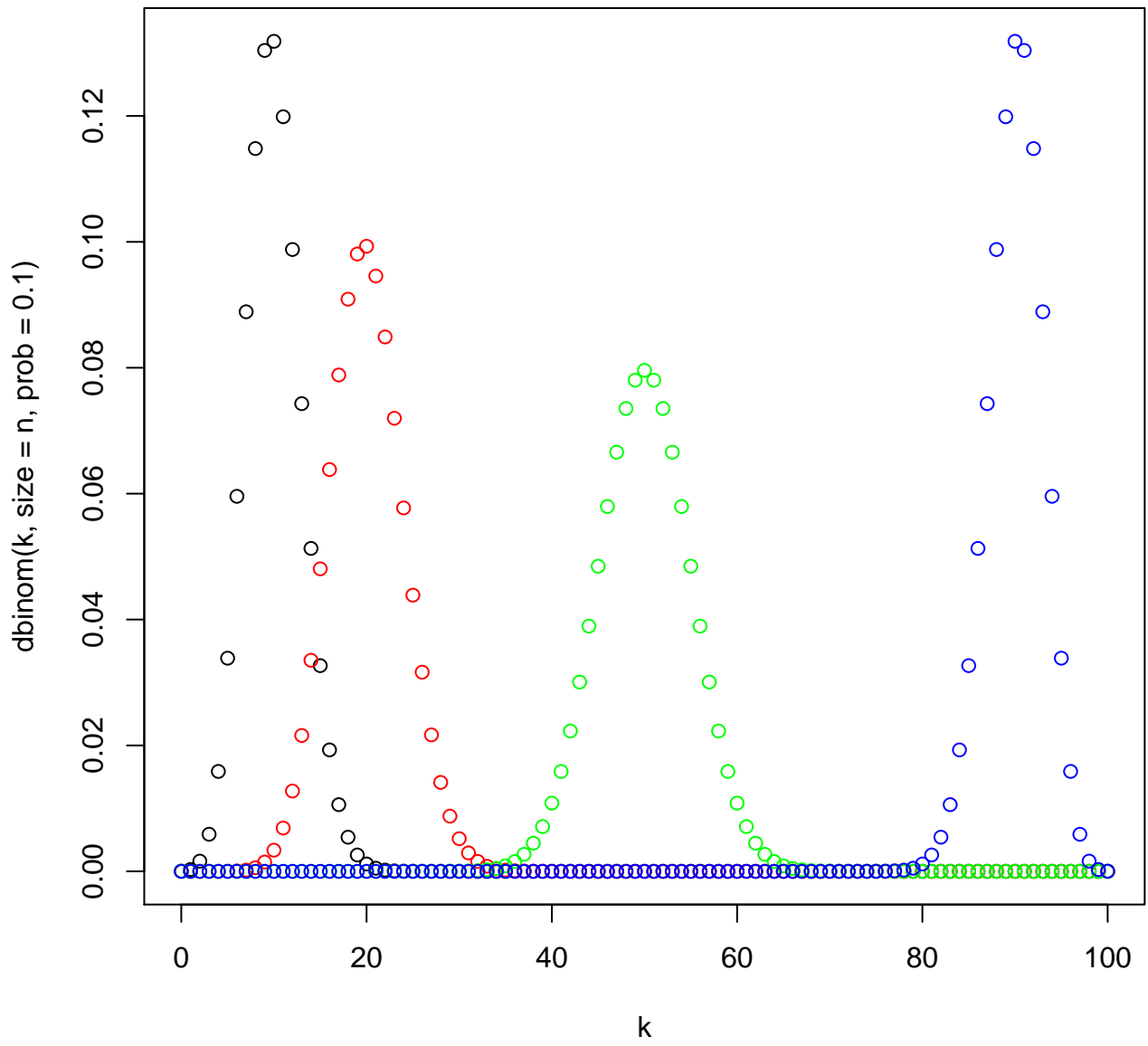


Figure 6: Graph for Problem 6

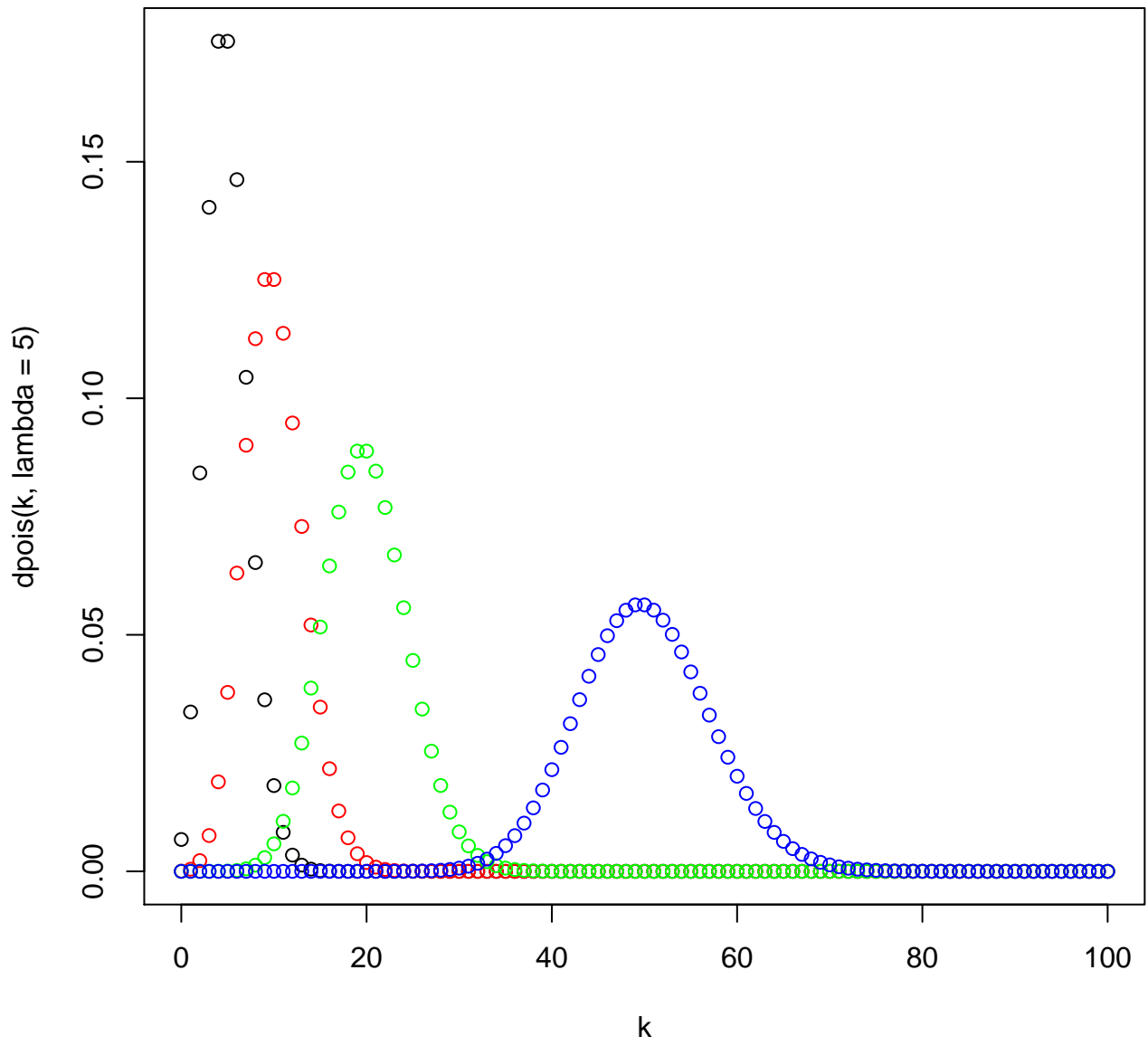


Figure 7: Graph for Problem 7