Ma 3520: Differential Equations and Dynamical Systems
Homework Assignment 1

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Read Chapters 1 and 2 of the textbook, "Nonlinear Dynamics and Chaos," third edition, by Steven H.Strogatz. Upload your complete solutions using GradeScope. **Late homework will not be accepted.**

Do the following exercises:

1. (Ex.2.1.2, p.38) At what points x does the flow $\dot{x} = \sin x$ have the greatest velocity to the right?
2. (Ex.2.2.3, p.39) For the equation $\dot{x} = x - x^3$, sketch the phase portrait, the vector field on the line, the fixed points classified by stability, and trajectories $x(t)$ for various initial values $x(0)$. Then solve the equation analytically.
3. (Ex.2.2.9, p.39) Find an equation $\dot{x} = f(x)$ whose trajectories resemble those in Fig.2 on p.40 of our textbook.
4. (Ex.2.3.1, p.42) Find the analytic solution to the logistic equation

$$\dot{N} = rN\left(1 - \frac{N}{K}\right); \qquad N(0) = N_0$$

for arbitrary initial values N_0 using these two methods:

(a) Use separation of variables.

(b) Substitute $N = 1/x$ and solve the resulting initial value problem for x.

5. (Ex.2.4.2, p.45) Use linear stability analysis to classify the fixed points of the system $\dot{x} = x(1-x)(2-x)$.
6. (Ex.2.4.7, p.45) Use linear stability analysis to classify the fixed points of the system $\dot{x} = ax - x^3$ for fixed $a \in \mathbf{R}$.
7. (Ex.2.5.2, p.46) Show that solutions to $\dot{x} = 1 + x^{10}$ blow up in finite time.
8. (Ex.2.5.4, p.46) Show that $\dot{x} = x^{1/3}$ has infinitely many solutions $x(t), t \geq 0$ satisfying $x(0) = 0$.
9. (Ex.2.7.3, p.48) Plot the potential function and classify the equilibrium points of $\dot{x} = \sin x$.
10. (Ex. 2.8.2, part(d), p.48) Sketch the slope field for $\dot{x} = \sin x$ and draw a few trajectories.
11. (Ex. 2.8.8, p.49) Use Taylor series to show that the improved Euler method (Heun's method) has local error $O(\Delta t^3)$.