Ma 3520: Differential Equations and Dynamical Systems
Homework Assignment 3

Prof. Wickerhauser

Read Chapters 5 and 6 of the textbook, "Nonlinear Dynamics and Chaos," 3rd ed., by Steven H. Strogatz. Upload your complete solutions using GradeScope. **Late homework will not be accepted.**

Do the following exercises:

1. (Ex.5.1.[3,4,5,6], p.155) Write the following linear systems in matrix form:
   (a) $\dot{x} = -y$, $\dot{y} = -x$.
   (b) $\dot{x} = 3x - 2y$, $\dot{y} = 2y - x$.
   (c) $\dot{x} = 0$, $\dot{y} = x + y$.
   (d) $\dot{x} = y$, $\dot{y} = 5x + y$.

2. (Ex.5.1.10, p.155) Read the definitions in the textbook's preamble to this exercise. Then for each of the following systems, decide whether the origin is attracting, Liapunov stable, asymptotically stable, or none of the above.
   (a) $\dot{x} = y$, $\dot{y} = -4x$.
   (b) $\dot{x} = 2y$, $\dot{y} = x$.
   (c) $\dot{x} = 0$, $\dot{y} = x$.
   (d) $\dot{x} = 0$, $\dot{y} = -y$.
   (e) $\dot{x} = -x$, $\dot{y} = -5y$.
   (f) $\dot{x} = x$, $\dot{y} = y$.

3. (Ex.5.2.[3,...,10], p.91) Plot the flow field in phase space and classify the fixed points of the following systems. If there are real eigenvectors, plot them on the diagram.
   [Hint: see `https://www.math.wustl.edu/~victor/classes/ma3520/lin2d.txt`]
   (a) $\dot{x} = y$, $\dot{y} = -2x - 3y$.
   (b) $\dot{x} = 5x + 10y$, $\dot{y} = -x - y$.
   (c) $\dot{x} = 3x - 4y$, $\dot{y} = x - y$.
   (d) $\dot{x} = -3x + 2y$, $\dot{y} = x - 2y$.
   (e) $\dot{x} = 5x + 2y$, $\dot{y} = -17x - 5y$.
   (f) $\dot{x} = -3x + 4y$, $\dot{y} = -2x + 3y$.
   (g) $\dot{x} = 4x - 3y$, $\dot{y} = 8x - 6y$.
   (h) $\dot{x} = y$, $\dot{y} = -x - 2y$.

4. (Ex.6.1.[8,9,10,11], p.199) Plot the flow fields in phase space for the following 2×2 dynamical systems: [Hint: see `https://www.math.wustl.edu/~victor/classes/ma3520/rk4a.txt`]

   (a) (Van der Pol's oscillator) $\dot{x} = y$, $\dot{y} = -x + y(1 - x^2)$.

   (b) (Dipole fixed point) $\dot{x} = 2xy$, $\dot{y} = y^2 - x^2$.

   (c) (Two-eyed monster) $\dot{x} = y + y^2$, $\dot{y} = -\frac{1}{2}x + \frac{1}{5}y - xy + \frac{6}{5}y^2$.

   (d) (Parrot) $\dot{x} = y + y^2$, $\dot{y} = -x + +\frac{1}{5}y - xy + \frac{6}{5}y^2$.

5. (Ex.6.3.10, p.201) Consider the system $\dot{x} = xy$, $\dot{y} = x^2 - y$, for which linearization is inconclusive.

   (a) Show that the linearization predicts that the origin is a non-isolated fixed point.

   (b) Show that the origin is in fact an isolated fixed point.

   (c) Classify the origin as attracting, repelling, saddle, or something else, using a sketch of the flow field along the nullclines.

   (d) Plot a computer-generated flow field phase portrait to confirm your answer to part (c).

6. (Ex.6.4.[1,2,3], p.203) For each of the following "rabbits vs. sheep" dynamical systems, where $x, y \geq 0$, find the fixed points, classify them by stability, draw the nullclines, and indicate the basins of attraction for any stable fixed points.

   (a) $\dot{x} = x(3 - x - y)$, $\dot{y} = y(2 - x - y)$.

   (b) $\dot{x} = x(3 - 2x - y)$, $\dot{y} = y(2 - x - y)$.

   (c) $\dot{x} = x(3 - 2x - 2y)$, $\dot{y} = y(2 - x - y)$.

7. (Ex.6.5.2, p.207) Consider the system $\ddot{x} = x - x^2$.

   (a) Find and classify the equilibrium points by stability.

   (b) Sketch the phase portrait flow field.

   (c) Find an equation for the homoclinic orbit that separates closed and nonclosed trajectories.

8. (Ex.6.5.4, p.207) Sketch the phase portrait for the system $\ddot{x} = ax - x^2$ for $a < 0$, $a = 0$, and $a > 0$.

9. (Ex.6.6.2, p.212) Show that the system $\dot{x} = y$, $\dot{y} = x \cos y$ is reversible, and sketch its phase portrait.

10. (Ex.6.6.10, p.214) Is the origin a nonlinear center for the system $\dot{x} = -y - x^2$, $\dot{y} = x$?