Ma 4102: Introduction to Lebesgue Integration
Final Examination Conditions and Problems List

Prof. Wickerhauser

10:30am-12:30pm on Monday, May 4th, in the classroom.

A small subset of the following problems will be on the final examination. No collaboration, electronic devices, books, notes, or other aids will be allowed.

1. State and prove Egoroff's theorem (30.1, p.85).
2. State and prove Lusin's theorem (30.3, p.87).
3. State the definition of a normed linear space and give both finite- and ∞-dimensional examples.
4. State the definition of a real Hilbert space and give both finite- and ∞-dimensional examples.
5. Determine whether $\{\cos kx : k = 0, 1, 2 \ldots\}$ is an orthogonal set in $\mathcal{L}^2[0, \pi]$.
6. State and prove Cauchy's inequality (32.13, p.98)
7. State and prove Minkowsky's inequality in $\mathcal{L}^2$ (33.9, p.104)
8. Define a trigonometric polynomial and a trigonometric series (p.113).
9. State and prove Bessel's inequality for an orthonormal set in $\mathcal{L}^2[-\pi, \pi]$.
10. State and prove the Riemann-Lebesgue lemma for the Fourier series of $f \in \mathcal{L}^2[-\pi, \pi]$.
11. Give two formulas for the Dirichlet kernel $D_n(t)$. (37.4, p.121)
12. Express the partial sum $s_n(x)$ of the Fourier series for a function $f \in \mathcal{L}^2[-\pi, \pi]$ using the Lebesgue integral and the Dirichlet kernel. (37.5, p.123)
13. Give a formula for the Fejér kernel $K_n(t)$ and relate it to the Dirichlet kernel. (37.6, p.124)
14. Define the Cesàro $(C, 1)$-sum of a series $\sum_{k=1}^{\infty} a_k$. Give an example of a divergent series with a convergent $(C, 1)$-sum.
15. Express the $(C, 1)$-sum $\sigma_n(x)$ of the Fourier series for a function $f \in \mathcal{L}[-\pi, \pi]$ using the Lebesgue integral and the Fejér kernel. (37.6, p.124)
16. Prove that if $f \in \mathcal{L}^2[-\pi, \pi]$ and f is continuous, then $\{\sigma_n\}$ converges uniformly to f. (37.8, p.125)
17. Prove that if $f \in \mathcal{L}^2[-\pi, \pi]$ and f is differentiable at x, then $\{s_n(x)\}$ converges to $f(x)$. (40.2, p.139)
18. State and prove the Riemann localization lemma (41.2, p.141)
19. Compute the complex exponential Fourier series of $f(x) = \sin x + 2 \cos 3x$.