Ma 416: Complex Variables
Homework Assignment 3

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Due Thursday, September 22nd, 2005

1. Find the Maclaurin series of $\sinh z = \frac{1}{2}(e^z - e^{-z})$.

2. Show that $\sinh z$ has infinitely many zeroes. (Hint: first express $\sinh z$ in terms of the sine function.)

3. If $a_n \geq 0$ and $\sum_{n=1}^{\infty} n a_n x^{n-1}$ converges for every $x \in [0, 1]$, prove that $\sum_{n=1}^{\infty} a_n x^{n-1}$ converges in the same interval.

4. Suppose that an analytic function f has arbitrarily small periods. That is, suppose that there is an infinite sequence $\{p_k : k \in \mathbf{N}\}$ with $|p_k| \to 0$ as $k \to \infty$ such that $f(z + p_k) = f(z)$ for all k and all $z \in \mathbf{C}$. Prove that f must be constant.

5. (a) Is there a solution $z \in \mathbf{C}$ to the equation $e^z = 0$? (b) Is there a solution $z \in \mathbf{C}$ to the equation $\tan z = i$?

6. Obtain formulas for the sums $\sin\theta + \sin 2\theta + \cdots + \sin n\theta$ and $1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta$ by considering the real and imaginary parts of the geometric series $\sum_{k=0}^{n} e^{ik\theta}$.

7. Let $C = \{z : |z| = r\}$ be a circle of radius $r > 0$, centered at the origin in $\mathbf{C}$, equipped with the positive (counterclockwise) orientation. (a) Compute $\int_C (1/z)\, dz$. (b) Compute $\int_C (1/\bar{z})\, dz$. (Hint: parametrize C.)

8. Find all the zeros of the function $f(z) = 2 + \cos z$. (Hint: if they exist, they must be nonreal.)