

Ma 416: Complex Variables

Homework Assignment 6

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Due Thursday, October 13th, 2005

Read R. P. Boas, *Invitation to Complex Analysis*, Chapter 2, sections 9A–11C.

1. Evaluate the definite integral $\int_0^{2\pi} (2 + \sin \theta)^{-2} d\theta$.
2. Evaluate the improper integral $\int_0^\infty (x^4 + 1)^{-1} dx$.
3. Evaluate the principal value integral $\text{PV} \int_{-\infty}^\infty (x^3 - 1)^{-1} dx$.
4. Evaluate the integral

$$\int_{-\infty}^\infty \frac{e^{x/n}}{1 - e^x} dx,$$

for $n = 1, 2, 3, \dots$ (Hint: use rectangular contours of fixed height 2π with one side lying on the x -axis.)

5. For each relation below, find all the complex numbers z satisfying it:
 - (a) $z^n = 1$ for a fixed $n \in \{2, 3, 4, \dots\}$
 - (b) $e^z = -e$
 - (c) $e^{\sqrt{z}} = i$
 - (d) $\tan z = 1 + i$.

6. Evaluate the integral

$$\int_0^\infty \frac{dx}{(1 + x^2)\sqrt{x}}.$$