Ma 416: Complex Variables
Homework Assignment 9

Prof. Wickerhauser

Due Thursday, November 10th, 2005

Read R. P. Boas, *Invitation to Complex Analysis*, Chapter 2, sections 16A–16C.

1. Suppose f is analytic on the closed unit disk, $f(0) = 0$, and $|f(z)| \leq |e^z|$ whenever $|z| = 1$. How big can $f((1+i)/2)$ be?

2. Prove Schwarz's lemma for a disk of radius R: If f is analytic on a closed disk D of radius R centered at z_0, $f(z_0) = 0$, and $|f(z)| \leq M$ on the boundary circle of D, then $|f(z)| \leq |z - z_0|M/R$ for each z inside D, with equality holding at some interior point z if and only if $f(z) = e^{ic}(z - z_0)$ for some constant $c \in \mathbf{R}$.

3. Use the radius-R Schwarz lemma of Problem 2 to prove Liouville's theorem. (Hint: apply the lemma to $f(z) - f(0)$.)

4. Prove that an entire function whose real part is bounded must be constant. (Hint: apply Liouville's theorem to the function e^f.)

5. Suppose that f is analytic on the closed unit disk, $f(0) = 0$, and $|\Re f(z)| \leq |e^z|$ for $|z| < 1$. Can $f((1+i)/2)$ be 18?