Ma 416: Complex Variables
Solutions to Homework Assignment 1

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Due Thursday, September 8th, 2005

1. Find the real parts, imaginary parts, and absolute values of the complex numbers

$$(a) \quad \frac{i+1}{i-1} \qquad (b) \quad \frac{1}{(1+2i)(3i-4)}$$

Solution: (a) real part 0, imaginary part -1, absolute value 1.
(b) real part $-2/25$, imaginary part $1/25$, absolute value $1/\sqrt{125} = 1/(5\sqrt{5})$. □

2. Graph the sets of points described by each of the following formulas:
(a) $|z-i| \le 2$
(b) Im $z > 2$ Re z

Solution: (a) This is a closed disk of radius 2 centered at $0 + i = (0, 1)$ in the complex plane.
(b) This is an open half-plane lying above the line $y = 2x$. □

3. Find the absolute value and principal argument for the following expressions:
(a) $3\,[\cos(2\pi/3) + i\sin(2\pi/3)]$
(b) $(3 + 4i)/(5i - 12)$

Solution: (a) This number is in polar form $r(\cos\theta + i\sin\theta)$ with an angle θ in the principal range $(-\pi, \pi]$, so we simply read the absolute value $r = 3$ and principal argument $2\pi/3$.

(b) Compute the absolute value as the ratio of the numerator and denominator absolute values: 5/13. Compute an argument from the complex ratio after eliminating the denominator: $(3+4i)/(5i-12) = (-16-63i)/169$, so we may use $\arctan(63/16) \approx 1.3221 \in (-\pi, \pi]$. Note that this is the same as the difference of the numerator and denominator principal arguments:

$$\arctan(4/3) - \arctan(-5/12) = \arctan(4/3) + \arctan(5/12) = \arctan(63/16),$$

though the difference of principal arguments may not fall in the range $(-\pi, \pi]$ in general. □

4. Find an argument in the interval $[0, 2\pi)$ for the following expressions, valid for any complex number z:
(a) $z - \bar{z}$
(b) $z + \bar{z}$
(c) $z\bar{z}$
(d) $z/\bar{z}$, if $z \neq 0$

Solution: (a) $\arg(z - \bar{z}) \in \{\pi/2, 3\pi/2\}$, since this difference is purely imaginary.

(b) $\arg(z + \bar{z}) \in \{0, \pi\}$, since this sum is purely real.

(c) $\arg(z\bar{z}) = \arg(|z|^2) = 0$, since the absolute value is purely real and positive.

(d) $\arg(z/\bar{z}) = \arg(z) - \arg(\bar{z}) = \arg(z) + \arg(z) = 2\arg(z)$, for any $z \neq 0$. This will be in the interval $[0, 2\pi)$ for $\arg(z) \in [0, \pi)$; if $\arg(z) \in [\pi, 2\pi)$, use $\arg(z/\bar{z}) = 2\arg(z) - 2\pi$. □

5. Simplify $(1+i)^{17}$ into the form $a+bi$.

Solution: Write $1+i=\sqrt{2}[\cos(\pi/4)+i\sin(\pi/4)]$ and use De Moivre's formula to obtain

$$(1+i)^{17}=2^{17/2}[\cos(17\pi/4)+i\sin(17\pi/4)]=256+256i.$$

□

6. Find all complex numbers z satisfying the equation $|z|^2=2\bar{z}$.

Solution: Write $z=r(\cos\theta+i\sin\theta)$ for real $r>0$ and $\theta\in[0,2\pi)$. The equation becomes

$$r^2=2r(\cos\theta-i\sin\theta),$$

which is evidently satisfied by $r=0$ and any θ, namely $z=0$, and also by the points with $r>0$ on the polar curve

$$r=2(\cos\theta-i\sin\theta).$$

But since r is purely real, we must have $\sin\theta=0$. Thus $\cos\theta=1$, so $r=2$ and $z=2+0i$.

Hence the only solutions are $z=0$ and $z=2$. □