1. Suppose that a risky asset \( S \) has spot price \( S(0) = 100 \) and that the riskless return to \( T = 1 \) year is \( R = 1.0537 \). Assuming there are no arbitrages, compute the following:
   (a) the current zero-coupon bond discount \( Z(0, T) \),
   (b) the Forward price for one share of \( S \) at expiry \( T \),
   (c) the riskless annual interest rate (assuming continuous compounding),

2. With \( S(0), R, \) and \( T \) as in Exercise 1, suppose that \( S(T) \) is modeled by

\[
\begin{array}{c|cccccccc}
S(T) & 93 & 95 & 98 & 104 & 107 & 110 & 114 \\
Pr(S(T)) & 0.02 & 0.05 & 0.09 & 0.29 & 0.31 & 0.19 & 0.05 \\
\end{array}
\]

(a) Use this finite probability space model to estimate premiums \( C(0) \) and \( P(0) \) for European-style Call and Put options, respectively, with strike price \( K = 101 \) and expiry \( T \).
(b) Does Call-Put Parity hold in this model? What might cause it to be inaccurate?

3. Use the no arbitrage Axiom 1 to prove that Eq.1.7 holds.

4. Prove Corollary 1.3. (Hint: review the proof of Theorem 1.2.)

5. Suppose, in contradiction with Eq.1.16, that \( C(0) - P(0) < S(0) - K/R \).
   Construct an arbitrage.

6. Prove Eq.1.20, the Call-Put parity Formula for foreign exchange options:

\[
C(0) - P(0) = \frac{X(0)}{R_f} - \frac{K}{R_d},
\]

using the no arbitrage axiom.

7. (a) Prove that the plus-part function satisfies Eq.1.17:

\[
[X]^+ - [-X]^+ = X,
\]

for any number \( X \).
(b) Apply the identity in part (a) to the payoff values of European-style Call and Put options for $S$ at strike price $K$ and expiry $T$ to show Eq. 1.18:

$$C(T) - P(T) = S(T) - K.$$ 

8. Plot the payoff and profit graphs for the following colorfully named option portfolios as a function of the price $S(T)$ at expiry time $T$:

(a) **Long straddle:** buy one Call and one Put on $S$ with the same expiry $T$ and at-the-money strike price $K \approx S(0)$. For what values of $S(T)$ will this be profitable?

(b) **Long strangle:** buy one Call at $K_c$ and one Put at $K_p$ with the same expiry $T$ but with out-of-the-money strike prices $K_p < S(0) < K_c$. How does its profitability compare with that of a long straddle?

9. A **butterfly spread** is a portfolio of European-style Call options purchased at time $t = 0$ with the same expiry $t = T$ but at three strike prices $L < M < H$, where $M = \frac{1}{2}(L + H)$:

- buy one Call $C_L$ at strike price $L$ for $C_L(0)$,
- buy one Call $C_H$ at strike price $H$ for $C_H(0)$,
- sell two Calls $C_M$ short at strike price $M$ for $2C_M(0)$.

(a) Plot the payoff graph for the butterfly spread at expiry when its price is $C_L(T) + C_H(T) - 2C_M(T)$. Mark the three strike prices on the $S(T)$ axis.

(b) Conclude from the graph for (a) that $C_M(0) < \frac{1}{2}[C_L(0) + C_H(0)]$. 
(Hint: use the no arbitrage Axiom 1.)

10. An **iron condor** is a portfolio $C_1 - C_2 - P_3 + P_4$ of four European-style options. To construct it, simultaneously buy one Call at $K_1$, sell one Call at $K_2$, sell one Put at $K_3$, and buy one Put at $K_4$, all with the same expiry $T$ but with $K_1 < K_2 < K_3 < K_4$.

(a) Plot or describe the payoff graph for an iron condor portfolio at expiry.

(b) Assuming no arbitrage, prove that the portfolio must have a positive net premium.

(c) Assuming no arbitrage, find inequalities bounding the maximum profit and the maximum loss of an iron condor portfolio at expiry.