

# Math 456: Introduction to Financial Mathematics

## Homework Set 4

1. Implement a compound Put option, the option to purchase a European-style Put option, with expiry  $T$  and strike price  $K$ , for price  $L$  at time  $T_1$  satisfying  $0 < T_1 < T$ . Use your code to price such an option with parameters  $(T, T_1, S_0, K, L, r, v, N) = (1, 0.5, 95, 99, 2.50, 0.04, 0.22, 20)$ . (Hint: modify `CRRcc.m`.)
2. Let  $\mathcal{M}(n)$  be the set of path numbers in a recombining binomial tree to depth  $n$ , as defined in Eq.4.6. Prove that

$$\mathcal{M}(n+1) = [2\mathcal{M}(n)] \cup [2\mathcal{M}(n) + 1],$$

where  $aX + b \stackrel{\text{def}}{=} \{ax + b : x \in X\}$  for sets  $X$  of numbers.

3. Implement floating strike option pricing in the CRR model using geometric means instead of arithmetic means, as in `CRRgro` versus `CRRaro`. Compare the results on the inputs  $(T, S_0, r, v, N) = (1.5, 90, 0.03, 0.21, 10)$ .
4. Use `CRRaro` to compute the average-rate Call and Put premiums in the CRR model for  $S_0 = K = 99$ ,  $T = 2$ ,  $r = 0.035$ ,  $v = 0.25$ , and different values of  $N$  such as  $N = 3, 5, 7, 9, 11$ . Compare  $\overline{C}(0) - \overline{P}(0)$  with the limit value in Eq.4.20.
5. Implement floating strike option pricing in the CRR model using path-dependent Arrow-Debreu securities. Check that the results agree with `CRRflt` on that function's suggested example inputs.
6. Write an Octave program to compute the maximums along all paths in a non-recombining binary tree of depth  $N$ . (Hint: modify `NRTmin()`.) Run both programs on the inputs `N=3` and

```
Sbar=[2 1 3 4 1 0 4 5 3 1 2 4 6 1 2]
```

Compare the results to confirm that path maxima are at least as large as path minima.

7. Implement lookback option pricing in the CRR model using path-dependent Arrow-Debreu securities. Check that the results agree with `CRRlb` on the inputs  $(T, S_0, r, v, N) = (1.5, 87, 0.033, 0.21, 8)$ .

8. Implement ladder Put option pricing in the CRR model by modifying `CRRladC` appropriately. Check that the premium is at least as great as that for the vanilla European-style Put with the same parameters, using the example inputs suggested within `CRRladC`.