Math 456: Introduction to Financial Mathematics

Homework Set 3

Due 23 October 2023

1. Suppose that $S(t, \omega)$, $0 \leq t \leq T$ is the price of a risky asset $S$, and that the riskless return over time $[0, T]$ is $R$. Model the future at time $t = T$ using $\Omega = \{\uparrow, \downarrow\}$ and assume that $S(T, \downarrow) < S(T, \uparrow)$.

(a) Use the no-arbitrage Axiom 1 to conclude that $S(T, \downarrow) < RS(0) < S(T, \uparrow)$.

(b) Use the Fair Price Theorem 1.4 to prove the same inequalities.

Solution: (a) First exclude the case $RS(0) \leq S(T, \downarrow) < S(T, \uparrow)$, as it offers the following arbitrage opportunity:

- At time $t = 0$, borrow $S(0)$ from the bank and buy one share of $S$. Initial cost is 0.
- At time $t = T$, sell $S$ for $S(t, \omega)$ and repay the loan with interest for $RS(0)$. The net result is $S(T, \omega) - RS(0)$.

The net at expiry is either $S(T, \downarrow) - RS(0) \geq 0$ or $S(T, \uparrow) - RS(0) > 0$, an arbitrage opportunity as claimed, forbidden by Axiom 1.

Second, exclude the possibility $S(T, \downarrow) < S(T, \uparrow) \leq RS(0)$ as it also provides an arbitrage opportunity:

- At time $t = 0$, sell $S$ short for $S(0)$ and deposit the money in the bank. Initial cost is again 0.
- At time $t = T$, withdraw the principal and interest $RS(0)$ from the bank and buy $S$ to cover the short for $S(t, \omega)$. The net result is $RS(0) - S(T, \omega)$.

The net at expiry in this case is either $RS(0) - S(T, \uparrow) \geq 0$ or $RS(0) - S(T, \downarrow) > 0$ giving an arbitrage opportunity as claimed, forbidden by Axiom 1.

Conclude that $S(T, \downarrow) < RS(0) < S(T, \uparrow)$.

(b) From Theorem 1.4 and the definition of expectation,

$$RS(0) = \Pr(\downarrow)S(T, \downarrow) + \Pr(\uparrow)S(T, \uparrow)$$
But $0 < \Pr(\downarrow) < 1$ since otherwise the future is certain, and thus also $0 < \Pr(\uparrow) < 1$ since $\Pr(\uparrow) = 1 - \Pr(\downarrow)$. Conclude that $RS(0)$ lies strictly inside the interval $[S(T, \downarrow), S(T, \uparrow)]$, as claimed. □

2. In Exercise 1 above, model the future at time $t = T$ using the $N$-step binomial model $\Omega = \{\omega_0, \omega_1, \ldots, \omega_N\}$ and assume that $S(T, \omega_k) = S(0)u^kd^{N-k}$, where $S(0) > 0$ is the spot price and $0 < d < u$ are the up factor and down factor, respectively, over one time step $T/N$.

(a) Use the no-arbitrage Axiom 1 to conclude that $d < R^{1/N} < u$.

(b) Use the Fair Price Theorem 1.4 to prove the same inequalities.

**Solution:** First note that $0 < d < u$ and $S(0) > 0$ together imply that $0 < S(T, \omega_0) = S(0)d^N < S(T, \omega_1) < \cdots < S(T, \omega_N) = S(0)u^N$.

(a) If $R^{1/N} \leq d < u$, then $R \leq d^N < u^N$, which implies

$$RS(0) \leq S(T, \omega_0); \quad RS(0) < S(T, \omega_k), \quad k = 1, \ldots, N,$$

so that borrowing $S(0)$ to buy $S$ at $t = 0$ will result in an arbitrage profit at $t = T$. Similarly, if $d < u \leq R^{1/N}$, then $d^N < u^N \leq R$, which implies

$$RS(0) \geq S(T, \omega_N); \quad RS(0) > S(T, \omega_k), \quad 0 \leq k < N,$$

so that selling $S$ short for $S(0)$ and investing the money risklessly at $t = 0$ will result in an arbitrage profit at $t = T$.

Both cases are forbidden by Axiom 1, so $d < R^{1/N} < u$.

(b) From Theorem 1.4 and the definition of expectation,

$$RS(0) = \mathbb{E}(S(T)) = \sum_{k=0}^{N} \Pr(\omega_k)S(T, \omega_k) = S(0)\sum_{k=0}^{N} \Pr(\omega_k)u^kd^{N-k}.$$

Dividing by $S(0) > 0$ simplifies this to

$$R = \sum_{k=0}^{N} \Pr(\omega_k)u^kd^{N-k}.$$

Now $0 < d < u$ implies that $d^N < ud^{N-1} < \cdots < u^{N-1}d < u^N$, and the probabilities lie in $[0, 1]$ and sum to 1, so the right-hand side is a convex combination of points in the interval $[d^N, u^N]$. Since the asset is risky, at least two of the states have positive probabilities. The convex combination must therefore lie strictly inside the interval, so

$$d^N < R < u^N,$$

from which it follows that $d < R^{1/N} < u$ as claimed. □
3. Suppose that a portfolio \( X \) contains risky stock \( S \) and riskless bond \( B \) in amounts \( h_0, h_1 \):

\[
X(t, \omega) = h_0 B(t, \omega) + h_1 S(t, \omega).
\]

Model the future at time \( t = T \) using \( \Omega = \{\uparrow, \downarrow\} \), assuming only that 
\( S(T, \uparrow) \neq S(T, \downarrow) \) and that \( B(T, \uparrow) = B(T, \downarrow) = R \). Compute \( h_0 \) and \( h_1 \) in terms of all the other quantities. (Hint: use Macsyma to derive Eq.3.1.)

**Solution:** Set up the system of equations at \( t = T \):

\[
\begin{align*}
X(T, \uparrow) &= h_0 B(T, \uparrow) + h_1 S(T, \uparrow) = h_0 R + h_1 S(T, \uparrow) \\
X(T, \downarrow) &= h_0 B(T, \downarrow) + h_1 S(T, \downarrow) = h_0 R + h_1 S(T, \downarrow)
\end{align*}
\]

Use these Macsyma commands to solve the system:

\[
\begin{align*}
eq 1: & \quad xTu=h0*R+h1*sTu; /* Up state equation */ \\
eq 2: & \quad xTd=h0*R+h1*sTd; /* Down state equation */ \\
h0h1: & \quad solve([eq1, eq2], [h0, h1]); /* Solve for h0, h1 */
\end{align*}
\]

That results in this output:

\[
[[h0=-sTd*xTu-sTu*xTd)/(sTu-sTd)*R), \\
h1=(xTu-xTd)/(sTu-sTd)]]
\]

Writing the Macsyma solution in the original notation gives

\[
h_0 = \frac{S(T, \uparrow)X(T, \downarrow) - S(T, \downarrow)X(T, \uparrow)}{(S(T, \uparrow) - S(T, \downarrow))R}; \quad h_1 = \frac{X(T, \uparrow) - X(T, \downarrow)}{S(T, \uparrow) - S(T, \downarrow)}.
\]

4. In Exercise 3 above, suppose that \( X \) is a European-style Call option for \( S \) with expiry \( T \) and strike price \( K \). Use the payoff formula \( X(T) = \lfloor S(T) - K \rfloor^+ \) in the equation for \( h_1 \) to prove that 

\[
0 \leq h_1 \leq 1.
\]

Conclude that, in this model of the future, a European-style Call option for \( S \) is equivalent to a portfolio containing part of a share of \( S \) plus or minus some cash.

**Solution:** Substitute the payoff formula into the equation for \( h_1 \), then add and subtract \( K \) in the denominator to get:

\[
h_1 = \frac{X(T, \uparrow) - X(T, \downarrow)}{S(T, \uparrow) - S(T, \downarrow)} = \frac{[S(T, \uparrow) - K]^+ - [S(T, \downarrow) - K]^+}{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)}.
\]

It may be assumed that \( S(T, \uparrow) > S(T, \downarrow) \), since the states can be switched without changing the value of \( h_1 \). Then there are three cases to consider:
Case 1: If $S(T, \uparrow) > S(T, \downarrow) > K$, then both plus-parts are positive, so

$$h_1 = \frac{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)}{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)} = 1.$$ 

Case 2: If $K \geq S(T, \uparrow) > S(T, \downarrow)$, then both plus-parts are zero, so

$$h_1 = 0 - 0 = 0.$$ 

Case 3: If $S(T, \uparrow) > K \geq S(T, \downarrow)$, then the first plus-part is positive but the second is zero, so

$$h_1 = \frac{(S(T, \uparrow) - K) - 0}{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)}.$$ 

But the denominator is $(S(T, \downarrow) - K) \leq 0$, which implies

$$(S(T, \uparrow) - K) - (S(T, \downarrow) - K) \geq (S(T, \uparrow) - K) > 0,$$

so the positive denominator is no smaller than the positive numerator, so $0 < h_1 \leq 1$.

Conclude that $0 \leq h_1 \leq 1$ in all cases. \qed

5. In Exercise 3 above, suppose that $X$ is a European-style Put option for $S$ with expiry $T$ and strike price $K$. Use the payoff formula $X(T) = [K - S(T)]^+$ in the equation for $h_1$ to prove that

$$-1 \leq h_1 \leq 0.$$ 

Conclude that, in this model of the future, a European-style Put option for $S$ is equivalent to a portfolio containing part of a share of $S$ sold short plus or minus some cash.

Solution: Substitute the payoff formula into the equation for $h_1$, multiply numerator and denominator by $-1$, and then add and subtract $K$ in the denominator to get:

$$h_1 = \frac{X(T, \uparrow) - X(T, \downarrow)}{S(T, \uparrow) - S(T, \downarrow)} = -\frac{[K - S(T, \uparrow)]^+ - [K - S(T, \downarrow)]^+}{(K - S(T, \uparrow)) - (K - S(T, \downarrow))}.$$ 

It may be assumed that $S(T, \uparrow) > S(T, \downarrow)$, since the states can be switched without changing the value of $h_1$. Then there are three cases to consider:

Case 1: If $K > S(T, \uparrow) > S(T, \downarrow)$, then both plus-parts are positive, so

$$h_1 = -\frac{(K - S(T, \uparrow)) - (K - S(T, \downarrow))}{(K - S(T, \uparrow)) - (K - S(T, \downarrow))} = -1.$$
Case 2: If $S(T,↑) > S(T,↓) \geq K$, then both plus-parts are zero, so

$$h_1 = -\frac{0 - 0}{(K - S(T,↑)) - (K - S(T,↓))} = 0.$$ 

Case 3: If $S(T,↑) \geq K > S(T,↓)$, then the first plus-part is zero but the second is positive, so

$$h_1 = -\frac{0 - (K - S(T,↓))}{(K - S(T,↑)) - (K - S(T,↓))} = \frac{(K - S(T,↓))}{(K - S(T,↑)) - (K - S(T,↓))}.$$ 

But the denominator is $(K - S(T,↑)) \leq 0$, which implies

$$(K - S(T,↑)) - (K - S(T,↓)) \leq -(K - S(T,↓)) < 0,$$

so the negative denominator has no smaller absolute value than the positive numerator, so $-1 \leq h_1 < 0$.

Conclude that $-1 \leq h_1 \leq 0$ in all cases.

Remark. An alternative proof uses the identity $y = [y]^+ - [-y]^+$ which is true for any $y$. Then

$$(S(T,ω) - K) = [S(T,ω) - K]^+ - [K - S(T,ω)]^+,$$

so

$$[K - S(T,ω)]^+ = [S(T,ω) - K]^+ - (S(T,ω) - K),$$

and thus

$$h_1 = \frac{[S(T,↑) - K]^+ - [S(T,↓) - K])^+}{(S(T,↑) - K) - (S(T,↓) - K)} - 1.$$

The result now follows from the Call $h_1$ inequalities. \qed

6. Suppose that $C(0)$ and $P(0)$ are the premiums for European-style Call and Put options, respectively, on an asset $S$ with the following parameters: expiry at $T = 1$ year, spot price $S(0) = 90$, strike price $K = 95$. Assume that the riskless annual percentage rate is $r = 0.02$, and the volatility for $S$ is $\sigma = 0.15$, and that these will remain constant from now until expiry.

(a) Use a LibreOffice Calc spreadsheet to implement the Cox-Ross-Rubinstein (CRR) model to compute $C(0)$ and $P(0)$ with $N = 10$ time steps, using the backward pricing formula in Eq.3.18. (Hint: compare output with CRReurAD() to check for bugs.)

(b) Use the Octave function CRReurAD() with $N = 10$, $N = 100$, and $N = 1000$ time steps to compute $C(0)$ and $P(0)$. 

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(c) Repeat part (b) with the Octave function \texttt{CRReur()} on p.88, again using \( N = 10 \), \( N = 100 \), and \( N = 1000 \) time steps to compute \( C(0) \) and \( P(0) \). Profile the time required to compute them, and compare the time and the output with that of \texttt{CRReurAD()}. 

(d) Compare the prices from parts (b) and (c). Is it justified to use \( N = 1000 \)? Is \( N = 10 \) sufficiently accurate?

\textbf{Solution:} (a) See the spreadsheet \texttt{CRR.ods} in the programs archive. With \( N = 10 \) time steps, rounding to five significant digits, it computes \( C(0) = $4.1733 \) and \( P(0) = $7.2922 \).

(b) Implement the program \texttt{CRReurAD()} on p.76, input the parameters, and compute the CRR approximations at \( N = 10 \), \( N = 100 \), and \( N = 1000 \) with the commands

\begin{verbatim}
T=1; S0=90; K=95; r=0.02; v=0.15;
[C0,P0]=CRReurAD(T,S0,K,r,v,10); C(1,1),P(1,1) % 4.1733, 7.2922
[C0,P0]=CRReurAD(T,S0,K,r,v,100); C(1,1),P(1,1) % 4.0572, 7.1761
[C0,P0]=CRReurAD(T,S0,K,r,v,1000); C(1,1),P(1,1) % 4.0555, 7.1744
\end{verbatim}

At \( N = 10 \) time steps it returns \( C(0) = $4.1733 \) and \( P(0) = $7.2922 \), in agreement with the spreadsheet. After \( N = 100 \) time steps, \( C(0) = $4.0572 \) and \( P(0) = $7.1761 \). After \( N = 1000 \) time steps, \( C(0) = $4.0555 \) and \( P(0) = $7.1744 \). In all cases the computations are almost instantaneous, requiring no noticeable time.

(c) The Octave program \texttt{CRReur()} on p.88 uses the backward induction formula to compute Call and Put premiums. It therefore fills two recombining binomial trees of depth \( N \) at a cost of \( O(N^2) \) compared with the \( O(N) \) cost of the Arrow-Debreu expansion method.

Input the parameters and compute the CRR approximations at \( N = 10 \), \( N = 100 \), and \( N = 1000 \) with the commands

\begin{verbatim}
T=1; S0=90; K=95; r=0.02; v=0.15;
[C,P]=CRReur(T,S0,K,r,v,10); C(1,1),P(1,1) % 4.1733, 7.2922
profile on; CRReur(T,S0,K,r,v,10); profshow % 0.013 seconds
[C,P]=CRReur(T,S0,K,r,v,100); C(1,1),P(1,1) % 4.0572, 7.1761
profile on; CRReur(T,S0,K,r,v,100); profshow % 1.05 seconds
[C,P]=CRReur(T,S0,K,r,v,1000); C(1,1),P(1,1) % 4.0555, 7.1744
profile on; CRReur(T,S0,K,r,v,1000); profshow % 103 seconds
\end{verbatim}

Note that the outputs are pairs of matrices, so that to get just the premiums it is necessary to extract just the \((1,1)\) element.

At \( N = 10 \) time steps it almost instantaneously returns \( C(0) = $4.1733 \) and \( P(0) = $7.2922 \), in agreement with the spreadsheet. Profiled time was 0.013 seconds.
With $N = 100$ time steps it very quickly computes $C(0) = 4.0572$ and $P(0) = 7.1761$. Profiled time was 1.05 seconds.

With $N = 1000$ time steps it takes a considerably longer time to compute $C(0) = 4.0555$ and $P(0) = 7.1744$. Profiled time was 102 seconds.

Note that the ratios of profiled times agree with the $O(N^2)$ complexity estimate.

(d) $N = 100$ seems justified since the prices are quite different from the $N = 10$ values. It seems unjustified to use $N = 1000$, which costs much more time and space (using the backward induction algorithm) but gives almost the same result as $N = 100$. □

7. Compare the prices from parts (a) and (b) of previous Exercise 6 with the Black-Scholes prices computed using Eqs. 2.25 and 2.26. Plot the logarithm of the differences against $\log N$ to estimate the rate of convergence. (Hint: Use the programs in Chapter 2, Section 2.4.)

**Solution:** Use the parameters from Exercise 6 in the Octave program `BS()` on p.34, as follows:

```octave
T=1; S0=90; K=95; r=0.02; v=0.15; [C0,P0]=BS(T,S0,K,r,v)
```

This returns $C_{BS} = C0 = 4.0548$ and $P_{BS} = P0 = 7.1736$. Now compute the logarithms of the differences as a function of $\log N$:

```octave
Ns=[10,100,1000]; log(Ns)
C0=4.0548; CCRR=[4.1733,4.0572,4.0555]; log(abs(CCRR-C0))
P0=7.1736; PCRR=[7.2922,7.1761,7.1744]; log(abs(PCRR-P0))
```

The output is tabulated below:

| $N$  | $\log N$ | $\log |C_{CRR} - C_{BS}|$ | $\log |P_{CRR} - P_{BS}|$ |
|------|----------|--------------------------|--------------------------|
| 10   | 2.3026   | -2.1328                  | -2.1320                  |
| 100  | 4.6052   | -6.0323                  | -5.9915                  |
| 1000 | 6.9078   | -7.2644                  | -7.1309                  |

Finally, generate the log-log plots:

```octave
plot(log(Ns),log(abs(CCRR-C0))); title("Call Difference"); xlabel("log N"); ylabel("log|C_{CRR}-C_{BS}|"); figure;
plot(log(Ns),log(abs(PCRR-P0))); title("Put Difference"); xlabel("log N"); ylabel("log|P_{CRR}-P_{BS}|");
```

The results may be seen in Figure 1. For both Call and Put differences, the graphs are close to lines of slope $-1$, suggesting that the difference between Black-Scholes and its $N$-step CRR approximation is $O(N^{-1})$. This may be quantified by regression using `polyfit(x,y,1)`:
Figure 1: (From Exercise 7) Log-log plots showing the differences between Black-Scholes prices and their $N$-step CRR approximations, for certain European-style Call and Put options, as a function of $N$.

polyfit(log(Ns),log(abs(CRR-C0)),1) \ % -1.114310 -0.011598
polyfit(log(Ns),log(abs(PCRR-P0)),1) \ % -1.085497 -0.085887

The first output number is the slope of the least-squares line fitting the data, in both cases close to $-1$. The second is the intercept; it is an estimate for the logarithm of the constant in the $O(N^{-1})$ rate.

**Remark.** Using only 5 significant digits introduces substantial round-off error at large $N$, where the differences are small. This is unavoidable since the parameters are only specified to 2 or 3 significant digits.

8. Derive 3.32 on p.79:

\[
q = \frac{1}{2} + \frac{r + \frac{\sigma^2}{2}}{2\sqrt{T/N}} + O\left(\frac{T}{N}\right).
\]

**Solution:** Recall that $q = (u - 1/R)/(u - 1/u)$. Using Taylor’s approximation in the numerator gives

\[
[1 + \sigma\sqrt{T/N} + \frac{\sigma^2}{2} T/N + O(\sqrt{T/N}^3)] - [1 - \frac{rT}{N} + O(\frac{T}{N})^2],
\]

while in the denominator it gives

\[
[1 + \sigma\sqrt{T/N} + \frac{\sigma^2}{2} T/N + O(\sqrt{T/N}^3)] - [1 - \sigma\sqrt{T/N} + \frac{\sigma^2}{2} T/N + O(\sqrt{T/N}^3)].
\]
Canceling terms and simplifying the ratio gives

\[ q = \frac{\sigma \sqrt{\frac{T}{N}} + \left( r + \frac{\sigma^2}{2} \right) \frac{T}{N} + O \left( \sqrt{\frac{T}{N}} \right) + O \left( \left[ \frac{T}{N} \right]^2 \right)}{2 \sigma \sqrt{\frac{T}{N}} + O \left( \sqrt{\frac{T}{N}} \right)} = \frac{1}{2} + \frac{r + \frac{\sigma^2}{2}}{2 \sigma} \sqrt{\frac{T}{N}} + O \left( \frac{T}{N} \right), \]

as claimed.

9. Use the CRR approximation with \( N = 4 \) to compute the European-style Call option premiums at several hundred equally spaced spot prices \( 75 \leq S_0 \leq 115 \), with expiry \( T = 1 \), strike \( K = 95 \), \( r = 0.02 \), and \( \sigma = 0.15 \).

(a) Plot the values against \( S_0 \).

(b) At what values of \( S_0 \) in that range does the graph appear to be nonsmooth?

(c) Compute the points of nondifferentiability for \( S_0 \) in [75, 115].

**Solution:** (a) Use `CRReurAD()` in the following Octave code:

```octave
T=1; K=95; r=0.02; v=0.15; N=4; m=401;
S=linspace(75,115,m); CS=zeros(size(S));
for i=1:m
    S0=S(i); [C0,P0]=CRReurAD(T,S(i),K,r,v,N); CS(i)=C0;
end
plot(S,CS); title("CRR with N=4");xlabel("S0");ylabel("C0");
```

See the result in Figure 2.

(b) The graph appears piecewise linear with joints \( \hat{S}_0 \in \{82, 95, 110\} \) where the Call premium is not differentiable with respect to \( S_0 \).

(c) Compute the joints, or points of nondifferentiability \( \hat{S}_0 \) nearest \( K \), using Eq.3.39 and \( j \in \{N/2 - 1, N/2, N/2 + 1\} = \{1, 2, 3\} \):

\[
\hat{S}_0 \in \left\{ \frac{K}{u^{2(j+1)-N}}, \frac{K}{u^{2j-N}}, \frac{K}{u^{2(j-1)-N}} \right\} = \{d^2K, K, u^2K\},
\]

where \( 1/d = u = \exp(\sigma \sqrt{T/N}) \). With the given parameters,

\[
u = \exp\left(0.15\sqrt{1/4}\right) = 1.0779, \quad \implies \hat{S}_0 \in \{81.767, 95, 110.37\},
\]

in good agreement with the visual estimate.

10. Compute the CRR option premiums and Greeks for European-style Call and Put options on a risky asset with the following parameters: spot price \$90, strike price \$95, expiry in 1 year, annual riskless rate 2%, and volatility 15%. Use \( N = 100 \) steps. Justify the method used.

**Solution:** First compute the option premiums with `CRReurAD()`:
Figure 2: (From Exercise 9) CRR approximation with \( N = 4 \) to the European-style Call option premium \( C(0) \), as a function of spot price \( S(0) \).

\[
T=1; \ S0=90; \ K=95; \ r=0.02; \ v=0.15; \ N=100;
\]

\[
[C0,P0]=
\]

For \( \Delta \) and \( \Gamma \), use the interpolation method on p.83:

\[
h0=2*S0*v*sqrt(T/N); \quad \% \text{critical h}
\]

\[
u2=exp(2*v*sqrt(T/N)); \quad \% \text{squared up factor}
\]

\[
x=[S0/u2, \ S0, \ S0*u2]-S0; \quad \% \text{shifted abscissas}
\]

\[
[C0,P0]=
\]

\[
[C0u,P0u]=
\]

\[
[C0d,P0d]=
\]

\[
yC=[C0d, \ C0, \ C0u]; \quad \% \text{Call ordinates}
\]

\[
p=polyfit(x,yC,2); \quad \text{DeltaC=p(2), GammaC=2*p(1)}
\]

\[
yP=[P0d, \ P0, \ P0u]; \quad \% \text{Put ordinates}
\]

\[
p=polyfit(x,yP,2); \quad \text{DeltaP=p(2), GammaP=2*p(1)}
\]

This is necessary because the approximation is not differentiable with respect to \( S_0 \).

For the other Greeks, use the centered difference approximation to the derivative with \( h \) set to 10% of the abscissa value:

\[
h=0.10*T; \quad \% \text{for ThetaC, ThetaP}
\]

\[
[C0u,P0u]=
\]

\[
[C0d,P0d]=
\]
ThetaC=-(C0u-C0d)/(2*h), ThetaP=-(P0u-P0d)/(2*h)

\[ h = 0.10*v; \] % for VegaC, VegaP
\[ [C0u,P0u] = CRReurAD(T,S0,K,r,v+h,N); \]
\[ [C0d,P0d] = CRReurAD(T,S0,K,r,v-h,N); \]
VegaC=(C0u-C0d)/(2*h), VegaP=(P0u-P0d)/(2*h)
\[ h = 0.10*r; \] % for RhoC, RhoP
\[ [C0u,P0u] = CRReurAD(T,S0,K,r+h,v,N); \]
\[ [C0d,P0d] = CRReurAD(T,S0,K,r-h,v,N); \]
RhoC=(C0u-C0d)/(2*h), RhoP=(P0u-P0d)/(2*h)

Since the centered difference approximation has \(O(h^2) \approx 1\%\) relative error, expect roughly two significant digits of accuracy. Much smaller values of \(h\) are not justified since the option premiums are only \(O(1/N) \approx 1\%\) accurate as shown in Exercise 7.

The results are tabulated below:

<table>
<thead>
<tr>
<th>Option</th>
<th>Delta</th>
<th>Gamma</th>
<th>Theta</th>
<th>Vega</th>
<th>Rho</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>4.0572</td>
<td>0.43987</td>
<td>-0.029063</td>
<td>36.298</td>
<td>35.457</td>
</tr>
<tr>
<td>Put</td>
<td>7.1761</td>
<td>-0.56013</td>
<td>0.029063</td>
<td>36.298</td>
<td>-57.661</td>
</tr>
</tbody>
</table>

Comparison with the Black-Scholes premiums and Greeks computed in Chapter 2, Exercise 10 shows good agreement.