Math 456: Introduction to Financial Mathematics

Homework Set 3

Due 23 October 2022

1. Suppose that $S(t, \omega), 0 \leq t \leq T$ is the price of a risky asset $S$, and that the riskless return over time $[0, T]$ is $R$. Model the future at time $t = T$ using $\Omega = \{\uparrow, \downarrow\}$ and assume that $S(T, \downarrow) < S(T, \uparrow)$.

(a) Use the no-arbitrage Axiom 1 to conclude that $S(T, \downarrow) < RS(0) < S(T, \uparrow)$.

(b) Use the Fair Price Theorem 1.4 to prove the same inequalities.

Solution: (a) First exclude the case $RS(0) \leq S(T, \downarrow) < S(T, \uparrow)$, as it offers the following arbitrage opportunity:

- At time $t = 0$, borrow $S(0)$ from the bank and buy one share of $S$. Initial cost is 0.
- At time $t = T$, sell $S$ for $S(t, \omega)$ and repay the loan with interest for $RS(0)$. The net result is $S(T, \omega) - RS(0)$.

The net at expiry is either $S(T, \downarrow) - RS(0) \geq 0$ or $S(T, \uparrow) - RS(0) > 0$, an arbitrage opportunity as claimed, forbidden by Axiom 1.

Second, exclude the possibility $S(T, \downarrow) < S(T, \uparrow) \leq RS(0)$ as it also provides an arbitrage opportunity:

- At time $t = 0$, sell $S$ short for $S(0)$ and deposit the money in the bank. Initial cost is again 0.
- At time $t = T$, withdraw the principal and interest $RS(0)$ from the bank and buy $S$ to cover the short for $S(t, \omega)$. The net result is $RS(0) - S(T, \omega)$.

The net at expiry in this case is either $RS(0) - S(T, \uparrow) \geq 0$ or $RS(0) - S(T, \downarrow) > 0$ giving an arbitrage opportunity as claimed, forbidden by Axiom 1.

Conclude that $S(T, \downarrow) < RS(0) < S(T, \uparrow)$.

(b) From Theorem 1.4 and the definition of expectation,

$$RS(0) = \Pr(\downarrow)S(T, \downarrow) + \Pr(\uparrow)S(T, \uparrow)$$
But \(0 < \Pr(\downarrow) < 1\) since otherwise the future is certain, and thus also \(0 < \Pr(\uparrow) < 1\) since \(\Pr(\uparrow) = 1 - \Pr(\downarrow)\). Conclude that \(RS(0)\) lies strictly inside the interval \([S(T, \downarrow), S(T, \uparrow)]\), as claimed. \(\square\)

2. In Exercise 1 above, model the future at time \(t = T\) using the \(N\)-step binomial model \(\Omega = \{\omega_0, \omega_1, \ldots, \omega_N\}\) and assume that \(S(T, \omega_k) = S(0)u^k d^{N-k}\), where \(S(0) > 0\) is the spot price and \(0 < d < u\) are the up factor and down factor, respectively, over one time step \(T/N\).

(a) Use the no-arbitrage Axiom 1 to conclude that \(d < R^{1/N} < u\).

(b) Use the Fair Price Theorem 1.4 to prove the same inequalities.

**Solution:** First note that \(0 < d < u\) and \(S(0) > 0\) together imply that \(0 < S(T, \omega_0) = S(0)d^N < S(T, \omega_1) < \cdots < S(T, \omega_N) = S(0)u^N\).

(a) If \(R^{1/N} \leq d < u\), then \(R \leq d^N < u^N\), which implies

\[
RS(0) \leq S(T, \omega_0); \quad RS(0) < S(T, \omega_k), \quad k = 1, \ldots, N,
\]

so that borrowing \(S(0)\) to buy \(S\) at \(t = 0\) will result in an arbitrage profit at \(t = T\). Similarly, if \(d < u \leq R^{1/N}\), then \(d^N < u^N \leq R\), which implies

\[
RS(0) \geq S(T, \omega_N); \quad RS(0) > S(T, \omega_k), \quad 0 \leq k < N,
\]

so that selling \(S\) short for \(S(0)\) and investing the money risklessly at \(t = 0\) will result in an arbitrage profit at \(t = T\). Both cases are forbidden by Axiom 1, so \(d < R^{1/N} < u\).

(b) From Theorem 1.4 and the definition of expectation,

\[
RS(0) = E(S(T)) = \sum_{k=0}^{N} \Pr(\omega_k)S(T, \omega_k) = S(0) \sum_{k=0}^{N} \Pr(\omega_k)u^k d^{N-k}.
\]

Dividing by \(S(0) > 0\) simplifies this to

\[
R = \sum_{k=0}^{N} \Pr(\omega_k)u^k d^{N-k}.
\]

Now \(0 < d < u\) implies that \(d^N < ud^{N-1} < \cdots < u^{N-1}d < u^N\), and the probabilities lie in \([0, 1]\) and sum to 1, so the right-hand side is a convex combination of points in the interval \([d^N, u^N]\). Since the asset is risky, at least two of the states have positive probabilities. The convex combination must therefore lie strictly inside the interval, so

\[
d^N < R < u^N,
\]

from which it follows that \(d < R^{1/N} < u\) as claimed. \(\square\)
3. Suppose that a portfolio $X$ contains risky stock $S$ and riskless bond $B$ in amounts $h_0, h_1$:

$$X(t, \omega) = h_0 B(t, \omega) + h_1 S(t, \omega).$$

Model the future at time $t = T$ using $\Omega = \{\uparrow, \downarrow\}$, assuming only that $S(T, \uparrow) \neq S(T, \downarrow)$ and that $B(T, \uparrow) = B(T, \downarrow) = R$. Compute $h_0$ and $h_1$ in terms of all the other quantities. (Hint: use Macsyma to derive Eq.3.1.)

**Solution:** Set up the system of equations at $t = T$:

$$X(T, \uparrow) = h_0 B(T, \uparrow) + h_1 S(T, \uparrow) = h_0 R + h_1 S(T, \uparrow)$$

$$X(T, \downarrow) = h_0 B(T, \downarrow) + h_1 S(T, \downarrow) = h_0 R + h_1 S(T, \downarrow)$$

Use these Macsyma commands to solve the system:

```macsyma
eq1: xTu=h0*R+h1*sTu; /* Up state equation */
eq2: xTd=h0*R+h1*sTd; /* Down state equation */
ho:h1: solve([eq1,eq2],[h0,h1]); /* Solve for h0,h1 */
```

That results in this output:

$$[h_0 = \frac{S(T, \uparrow) X(T, \downarrow) - S(T, \downarrow) X(T, \uparrow)}{(S(T, \uparrow) - S(T, \downarrow)) R},$$

$$h_1 = \frac{X(T, \uparrow) - X(T, \downarrow)}{S(T, \uparrow) - S(T, \downarrow)}.$$

4. In Exercise 3 above, suppose that $X$ is a European-style Call option for $S$ with expiry $T$ and strike price $K$. Use the payoff formula $X(T) = [S(T) - K]^+$ in the equation for $h_1$ to prove that

$$0 \leq h_1 \leq 1.$$
Case 1: If $S(T, \uparrow) > S(T, \downarrow) > K$, then both plus-parts are positive, so

$$h_1 = \frac{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)}{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)} = 1.$$ 

Case 2: If $K \geq S(T, \uparrow) > S(T, \downarrow)$, then both plus-parts are zero, so

$$h_1 = \frac{0 - 0}{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)} = 0.$$ 

Case 3: If $S(T, \uparrow) > K \geq S(T, \downarrow)$, then the first plus-part is positive but the second is zero, so

$$h_1 = (S(T, \uparrow) - K) - 0 = S(T, \uparrow) - K - (S(T, \downarrow) - K) = 1.$$

But the denominator is $(S(T, \downarrow) - K) \leq 0$, which implies

$$(S(T, \uparrow) - K) - (S(T, \downarrow) - K) \geq (S(T, \uparrow) - K) > 0,$$

so the positive denominator is no smaller than the positive numerator, so $0 < h_1 \leq 1$.

Conclude that $0 \leq h_1 \leq 1$ in all cases.

5. In Exercise 3 above, suppose that $X$ is a European-style Put option for $S$ with expiry $T$ and strike price $K$. Use the payoff formula $X(T) = [K - S(T)]^+$ in the equation for $h_1$ to prove that

$$-1 \leq h_1 \leq 0.$$

Conclude that, in this model of the future, a European-style Put option for $S$ is equivalent to a portfolio containing part of a share of $S$ sold short plus or minus some cash.

Solution: Substitute the payoff formula into the equation for $h_1$, multiply numerator and denominator by $-1$, and then add and subtract $K$ in the denominator to get:

$$h_1 = \frac{X(T, \uparrow) - X(T, \downarrow)}{S(T, \uparrow) - S(T, \downarrow)} = -\frac{[K - S(T, \uparrow)]^+ - [K - S(T, \downarrow)]^+}{(K - S(T, \uparrow)) - (K - S(T, \downarrow))}.$$

It may be assumed that $S(T, \uparrow) > S(T, \downarrow)$, since the states can be switched without changing the value of $h_1$. Then there are three cases to consider:

Case 1: If $K > S(T, \uparrow) > S(T, \downarrow)$, then both plus-parts are positive, so

$$h_1 = -\frac{(K - S(T, \uparrow)) - (K - S(T, \downarrow))}{(K - S(T, \uparrow)) - (K - S(T, \downarrow))} = -1.$$
Case 2: If \( S(T, \uparrow) > S(T, \downarrow) \geq K \), then both plus-parts are zero, so
\[
h_1 = -\frac{0 - 0}{(K - S(T, \uparrow)) - (K - S(T, \downarrow))} = 0.
\]

Case 3: If \( S(T, \uparrow) \geq K > S(T, \downarrow) \), then the first plus-part is zero but the second is positive, so
\[
h_1 = -\frac{0 - (K - S(T, \downarrow))}{(K - S(T, \uparrow)) - (K - S(T, \downarrow))} = \frac{(K - S(T, \downarrow))}{(K - S(T, \uparrow)) - (K - S(T, \downarrow))}.
\]

But the denominator is \((K - S(T, \uparrow)) \leq 0\), which implies
\[
(K - S(T, \uparrow)) - (K - S(T, \downarrow)) \leq -(K - S(T, \downarrow)) < 0,
\]
so the negative denominator has no smaller absolute value than the positive numerator, so \(-1 \leq h_1 < 0\).

Conclude that \(-1 \leq h_1 \leq 0\) in all cases.

Remark. An alternative proof uses the identity \( y = [y]^+ - [-y]^+ \) which is true for any \( y \). Then
\[
(S(T, \omega) - K) = [S(T, \omega) - K]^+ - [K - S(T, \omega)]^+,
\]
so
\[
[K - S(T, \omega)]^+ = [S(T, \omega) - K]^+ - (S(T, \omega) - K),
\]
and thus
\[
h_1 = \frac{[S(T, \uparrow) - K]^+ - [S(T, \downarrow) - K])^+}{(S(T, \uparrow) - K) - (S(T, \downarrow) - K)} - 1.
\]
The result now follows from the Call \( h_1 \) inequalities. \( \square \)

6. Suppose that \( C(0) \) and \( P(0) \) are the premiums for European-style Call and Put options, respectively, on an asset \( S \) with the following parameters:
expiry at \( T = 1 \) year, spot price \( S(0) = 90 \), strike price \( K = 95 \). Assume that the riskless annual percentage rate is \( r = 0.02 \), and the volatility for \( S \) is \( \sigma = 0.15 \), and that these will remain constant from now until expiry.

(a) Use a LibreOffice Calc spreadsheet to implement the Cox-Ross-Rubinstein (CRR) model to compute \( C(0) \) and \( P(0) \) with \( N = 10 \) time steps, using the backward pricing formula in Eq.3.18. (Hint: compare output with \texttt{CRReurAD()} to check for bugs.)

(b) Use the Octave function \texttt{CRReurAD()} with \( N = 10 \), \( N = 100 \), and \( N = 1000 \) time steps to compute \( C(0) \) and \( P(0) \).
(c) Repeat part (b) with the Octave function \texttt{CRReur()} on p.88, again using $N = 10$, $N = 100$, and $N = 1000$ time steps to compute $C(0)$ and $P(0)$. Profile the time required to compute them, and compare the time and the output with that of \texttt{CRReurAD()}.

(d) Compare the prices from parts (b) and (c). Is it justified to use $N = 1000$? Is $N = 10$ sufficiently accurate?

\textbf{Solution:} (a) See the spreadsheet \texttt{CRR.ods} in the programs archive. With $N = 10$ time steps, rounding to five significant digits, it computes $C(0) = 4.1733$ and $P(0) = 7.2922$.

(b) Implement the program \texttt{CRReurAD()} on p.76, input the parameters, and compute the CRR approximations at $N = 10$, $N = 100$, and $N = 1000$ with the commands

\begin{verbatim}
T=1; S0=90; K=95; r=0.02; v=0.15;
[C0,P0]=CRReurAD(T,S0,K,r,v,10) % 4.1733, 7.2922
[C0,P0]=CRReurAD(T,S0,K,r,v,100) % 4.0572, 7.1761
[C0,P0]=CRReurAD(T,S0,K,r,v,1000) % 4.0555, 7.1744
\end{verbatim}

At $N = 10$ time steps it returns $C(0) = 4.1733$ and $P(0) = 7.2922$, in agreement with the spreadsheet.

After $N = 100$ time steps, $C(0) = 4.0572$ and $P(0) = 7.1761$.

After $N = 1000$ time steps, $C(0) = 4.0555$ and $P(0) = 7.1744$.

In all cases the computations are almost instantaneous, requiring no noticeable time.

(c) The Octave program \texttt{CRReur()}m on p.88 uses the backward induction formula to compute Call and Put premiums. It therefore fills two recombining binomial trees of depth $N$ at a cost of $O(N^2)$ compared with the $O(N)$ cost of the Arrow-Debreu expansion method.

Input the parameters and compute the CRR approximations at $N = 10$, $N = 100$, and $N = 1000$ with the commands

\begin{verbatim}
T=1; S0=90; K=95; r=0.02; v=0.15;
[C,P]=CRReur(T,S0,K,r,v,10); C(1,1),P(1,1) % 4.1733, 7.2922
profile on; CRReur(T,S0,K,r,v,10); profshow % -0.013 seconds
[C,P]=CRReur(T,S0,K,r,v,100); C(1,1),P(1,1) % 4.0572, 7.1761
profile on; CRReur(T,S0,K,r,v,100); profshow % -1.05 seconds
[C,P]=CRReur(T,S0,K,r,v,1000); C(1,1),P(1,1) % 4.0555, 7.1744
profile on; CRReur(T,S0,K,r,v,1000); profshow % -103 seconds
\end{verbatim}

Note that the outputs are pairs of matrices, so that to get just the premiums it is necessary to extract just the $(1,1)$ element.

At $N = 10$ time steps it almost instantaneously returns $C(0) = 4.1733$ and $P(0) = 7.2922$, in agreement with the spreadsheet. Profiled time was 0.013 seconds.
With $N = 100$ time steps it very quickly computes $C(0) = 4.0572$ and $P(0) = 7.1761$. Profiled time was 1.05 seconds.

With $N = 1000$ time steps it takes a considerably longer time to compute $C(0) = 4.0555$ and $P(0) = 7.1744$. Profiled time was 102 seconds.

Note that the ratios of profiled times agree with the $O(N^2)$ complexity estimate.

(d) $N = 100$ seems justified since the prices are quite different from the $N = 10$ values. It seems unjustified to use $N = 1000$, which costs much more time and space (using the backward induction algorithm) but gives almost the same result as $N = 100$. $\square$

7. Compare the prices from parts (a) and (b) of previous Exercise 6 with the Black-Scholes prices computed using Eqs.2.25 and 2.26. Plot the logarithm of the differences against $\log N$ to estimate the rate of convergence. (Hint: Use the programs in Chapter 2, Section 2.4.)

**Solution:** Use the parameters from Exercise 6 in the Octave program BS() on p.34, as follows:

```matlab
T=1; S0=90; K=95; r=0.02; v=0.15; [C0,P0]=BS(T,S0,K,r,v)
```

This returns $C_{BS} = C0 = 4.0548$ and $P_{BS} = P0 = 7.1736$. Now compute the logarithms of the differences as a function of $\log N$:

```matlab
Ns=[10,100,1000]; log(Ns)
C0=4.0548; CCRR=[4.1733,4.0572,4.0555]; log(abs(CCRR-C0))
P0=7.1736; PCRR=[7.2922,7.1761,7.1744]; log(abs(PCRR-P0))
```

The output is tabulated below:

| $N$  | $\log N$ | $\log |C_{CRR} - C_{BS}|$ | $\log |P_{CRR} - P_{BS}|$ |
|------|----------|---------------------------|---------------------------|
| 10   | 2.3026   | -2.1328                   | -2.1320                   |
| 100  | 4.6052   | -6.0323                   | -5.9915                   |
| 1000 | 6.9078   | -7.2644                   | -7.1309                   |

Finally, generate the log-log plots:

```matlab
plot(log(Ns),log(abs(CCRR-C0))); title("Call Difference");
xlabel("log N"); ylabel("log|C_{CRR}(N)-BS|"); figure;
plot(log(Ns),log(abs(PCRR-P0))); title("Put Difference");
xlabel("log N"); ylabel("log|P_{CRR}(N)-BS|");
```

The results may be seen in Figure 1. For both Call and Put differences, the graphs are close to lines of slope $-1$, suggesting that the difference between Black-Scholes and its $N$-step CRR approximation is $O(N^{-1})$. This may be quantified by regression using `polyfit(x,y,1):`
Figure 1: (From Exercise 7) Log-log plots showing the differences between Black-Scholes prices and their $N$-step CRR approximations, for certain European-style Call and Put options, as a function of $N$.

polyfit(log(Ns),log(abs(CRR-BS)),1) \% -1.114310 -0.011598
polyfit(log(Ns),log(abs(PCRR-PO)),1) \% -1.085497 -0.085887

The first output number is the slope of the least-squares line fitting the data, in both cases close to $-1$. The second is the intercept; it is an estimate for the logarithm of the constant in the $O(N^{-1})$ rate.

**Remark.** Using only 5 significant digits introduces substantial round-off error at large $N$, where the differences are small. This is unavoidable since the parameters are only specified to 2 or 3 significant digits.

8. Derive 3.32 on p.79:

$$ q = \frac{1}{2} + \frac{r + \sigma^2}{2 \sigma^2} \sqrt{\frac{T}{N}} + O\left(\frac{T}{N}\right). $$

**Solution:** Recall that $q = (u - 1/R)/(u - 1/u)$. Using Taylor’s approximation in the numerator gives

$$ [1 + \sigma \sqrt{\frac{T}{N}} + \frac{\sigma^2 T}{2 N} + O(\sqrt{\frac{T}{N}}^3)] - [1 - \frac{rT}{N} + O(\frac{T}{N})], $$

while in the denominator it gives

$$ [1+\sigma \sqrt{\frac{T}{N}} + \frac{\sigma^2 T}{2 N} + O(\sqrt{\frac{T}{N}}^3)] - [1-\sigma \sqrt{\frac{T}{N}} + \frac{\sigma^2 T}{2 N} + O(\sqrt{\frac{T}{N}}^3)]. $$
Canceling terms and simplifying the ratio gives

\[
q = \frac{\sigma \sqrt{\frac{T}{N}} + \left( r + \frac{\sigma^2}{2} \right) \frac{T}{N} + O \left( \sqrt{\frac{T}{N}} \right) + O \left( \left( \frac{T}{N} \right)^2 \right)}{2\sigma \sqrt{\frac{T}{N}} + O \left( \sqrt{\frac{T}{N}} \right)}
\]

\[
= \frac{1}{2} \left( 1 + \frac{r + \frac{\sigma^2}{2}}{2\sigma} \sqrt{\frac{T}{N}} + O \left( \frac{T}{N} \right) \right),
\]
as claimed. □

9. Use the CRR approximation with \( N = 4 \) to compute the European-style Call option premiums at several hundred equally spaced spot prices \( 75 \leq S_0 \leq 115 \), with expiry \( T = 1 \), strike \( K = 95 \), \( r = 0.02 \), and \( \sigma = 0.15 \).

(a) Plot the values against \( S_0 \).

(b) At what values of \( S_0 \) in that range does the graph appear to be nonsmooth?

(c) Compute the points of nondifferentiability for \( S_0 \) in \([75, 115]\).

**Solution:**

(a) Use `CRReurAD()` in the following Octave code:

```octave
T=1; K=95; r=0.02; v=0.15; N=4; m=401;
S=linspace(75,115,m); CS=zeros(size(S));
for i=1:m
    S0=S(i);
    [C0,P0]=CRReurAD(T,S0,K,r,v,N);
    CS(i)=C0;
end
plot(S,CS); title("CRR with N=4");xlabel("S0");ylabel("C0");
```

See the result in Figure 2.

(b) The graph appears piecewise linear with joints \( \hat{S}_0 \in \{82, 95, 110\} \) where the Call premium is not differentiable with respect to \( S_0 \).

(c) Compute the joints, or points of nondifferentiability \( \hat{S}_0 \) nearest \( K \), using Eq.3.39 and \( j \in \{N/2 - 1, N/2, N/2 + 1\} = \{1, 2, 3\} \):

\[
\hat{S}_0 \in \left\{ \frac{K}{u^{2(j+1)-N}}, \frac{K}{u^{2j-N}}, \frac{K}{u^{2(j-1)-N}} \right\} = \left\{ d^2 K, K, u^2 K \right\},
\]

where \( \frac{1}{d} = u = \exp(\sigma \sqrt{\frac{T}{N}}) \). With the given parameters,

\[
u = \exp(0.15 \sqrt{1/4}) = 1.0779, \quad \hat{S}_0 \in \{81.767, \ 95, \ 110.37\},
\]
in good agreement with the visual estimate. □

10. Compute the CRR option premiums and Greeks for European-style Call and Put options on a risky asset with the following parameters: spot price $90, strike price $95, expiry in 1 year, annual riskless rate 2%, and volatility 15%. Use \( N = 100 \) steps. Justify the method used.

**Solution:** First compute the option premiums with `CRReurAD()`:

```octave
```
Figure 2: (From Exercise 9) CRR approximation with $N = 4$ to the European-style Call option premium $C(0)$, as a function of spot price $S(0)$.

$T=1; S_0=90; K=95; r=0.02; \nu=0.15; N=100; 
[C_0, P_0] = \text{CRReurAD}(T, S_0, K, r, \nu, N)$

For $\Delta$ and $\Gamma$, use the interpolation method on p.83:

$h_0=2*S_0*\nu*\text{sqrt}(T/N); \% \text{critical } h$
$u_2=\exp(2*\nu*\text{sqrt}(T/N)); \% \text{squared up factor}$
$x=[S_0/u_2, S_0, S_0*u_2]-S_0; \% \text{shifted abscissas}$
$[C_0, P_0] = \text{CRReurAD}(T, S_0, K, r, \nu, N);$  
$[C_{0u}, P_{0u}] = \text{CRReurAD}(T, S_0*u_2, K, r, \nu, N); \% \text{at } S_0*u_2^2$
$[C_{0d}, P_{0d}] = \text{CRReurAD}(T, S_0/u_2, K, r, \nu, N); \% \text{at } S_0/u_2^2$
$y_C=[C_{0d}, C_0, C_{0u}]; \% \text{Call ordinates}$
$p=polyfit(x,y_C,2); \text{Delta}_C=p(2), \text{Gamma}_C=2*p(1)$
$y_P=[P_{0d}, P_0, P_{0u}]; \% \text{Put ordinates}$
$p=polyfit(x,y_P,2); \text{Delta}_P=p(2), \text{Gamma}_P=2*p(1)$

This is necessary because the approximation is not differentiable with respect to $S_0$.

For the other Greeks, use the centered difference approximation to the derivative with $h$ set to 10% of the abscissa value:

$h=0.10*T; \% \text{for } \Theta_C, \Theta_P$
$[C_{0u}, P_{0u}] = \text{CRReurAD}(T+h, S_0, K, r, \nu, N);$  
$[C_{0d}, P_{0d}] = \text{CRReurAD}(T-h, S_0, K, r, \nu, N);$
\[ \text{ThetaC} = \frac{(C_{0u} - C_{0d})}{2h}, \quad \text{ThetaP} = \frac{(P_{0u} - P_{0d})}{2h} \]

\[ h = 0.10 \times v; \quad \% \text{ for VegaC, VegaP} \]

\[ [C_{0u}, P_{0u}] = \text{CRReurAD}(T, S_0, K, r, v+h, N); \]

\[ [C_{0d}, P_{0d}] = \text{CRReurAD}(T, S_0, K, r, v-h, N); \]

\[ \text{VegaC} = \frac{(C_{0u} - C_{0d})}{2h}, \quad \text{VegaP} = \frac{(P_{0u} - P_{0d})}{2h} \]

\[ h = 0.10 \times r; \quad \% \text{ for RhoC, RhoP} \]

\[ [C_{0u}, P_{0u}] = \text{CRReurAD}(T, S_0, K, r+h, v, N); \]

\[ [C_{0d}, P_{0d}] = \text{CRReurAD}(T, S_0, K, r-h, v, N); \]

\[ \text{RhoC} = \frac{(C_{0u} - C_{0d})}{2h}, \quad \text{RhoP} = \frac{(P_{0u} - P_{0d})}{2h} \]

Since the centered difference approximation has \( O(h^2) \approx 1\% \) relative error, expect roughly two significant digits of accuracy. Much smaller values of \( h \) are not justified since the option premiums are only \( O(1/N) \approx 1\% \) accurate as shown in Exercise 7.

The results are tabulated below:

<table>
<thead>
<tr>
<th>Option</th>
<th>Delta</th>
<th>Gamma</th>
<th>Theta</th>
<th>Vega</th>
<th>Rho</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>4.0572</td>
<td>0.43987</td>
<td>0.029063</td>
<td>-3.4348</td>
<td>36.298</td>
</tr>
<tr>
<td>Put</td>
<td>7.1761</td>
<td>-0.56013</td>
<td>0.029063</td>
<td>-1.5724</td>
<td>36.298</td>
</tr>
</tbody>
</table>

Comparison with the Black-Scholes premiums and Greeks computed in Chapter 2, Exercise 10 shows good agreement. \( \square \)