Adapted waveform analysis as a tool for modeling, feature extraction and denoising.

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We describe the development of Adapted Waveform Analysis (AWA) as a tool for fast processing of the various identification tasks involved in medical diagnostics and Automatic Target Recognition.

These tasks typically consists of various stages:

i) Sensing and data acquisition

ii) Preprocessing for clutter and noise elimination as well as enhancement

iii) Parameter extraction

iv) Modelling

v) Classification

AWA is a tool for ameliorating each of these steps, either by accelerating the computation or by providing new means of analysis and modeling, for extracting features and classification.

Until recently, one of the main tools for processing measured signals has been the Fast Fourier Transform computed on segments of the signal.

In essence, this analysis consists in matching the signal to predetermined segments of (windowed) cosine waveforms.

AWA extends this analysis to a broader collection of waveforms (libraries), where the choice of waveforms for analysis (or appropriate orthonormal basis)is made automatically by a measure of fit between the class of "targets" and the corresponding waveforms.

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The Karhunen Loeve basis has been used in the past to achieve a similar goal, unfortunately K-L bases involve heavy computations both initially and in processing new acquired data,and only provide a decorrelated basis. AWA consists of fixed libraries giving rise to K-L-like bases equipped with fast numerical transforms (with roughly the same speed as FFT). AWA enables us to optimize the coordinate system used to represent the signal .Moreover this optimization can be adjusted to different goals by using various "cost functions".

The simplest description of these methods is provided by the following analogy. Consider the problem of speech storage . Clearly the least efficient way is to record the speech and digitize each sample (say 8000 per second). A most efficient method ,on the other hand ,is to transcribe the speech into english since each word is described by a few characters. Of course we keep the meaning but lose the voice, intonation etc. A more abstract and flexible method for audio compression (or description) is used by musicians who transcribe their music into musical scores. The score is a list of notes to be played at different times at different intensities and durations . For an orchestra the score will also specify an instrument. To carry this analogy further into images we might want to describe a painting as a superposition of brush stokes listed by layers. AWA is a mathematical version of the transcriptions described above with an essential added ingredient. The sound or images are analysed in realtime and a most efficient transription is found,(the minimal number of brushstrokes or the shortest score) . This transcription is lossless ,if so desired ,or could lose undesirable or irrelevent features if needed. Clearly the process of transcription provides a rudimentary model of the measured signal , permitting the extraction of few relevant parameter to feed into classifiers. The parameters obtained are more stable, giving more confidence.

Definitions of Modulated Waveform Libraries.

We start by recalling the concept of a "Library of orthonormal bases". For the sake of exposition we restrict our attention to two classes of numerically useful waveforms introduced recently [1][3].

We start with trigonometric waveform libraries. These are localized sine transforms LST associated to covering by intervals of $\mathbf{R}$ (more generally, of a manifold).

We consider a cover $\mathbf{R} = \bigcup_{-\infty}^{\infty} I_i \; I = [\alpha_i \alpha_{i+1}) \; \alpha_i < \alpha_{i+1}$, write $\ell_i = \alpha_{i+1} - \alpha_i = |I_i|$ and let $p_i(x)$ be a window function supported in $[\alpha_i - \ell_{i-1}/2, \alpha_{i+1} + \ell_{i+1}/2]$ such that

$$\sum_{-\infty}^{\infty} p_i^2(x) = 1$$

and

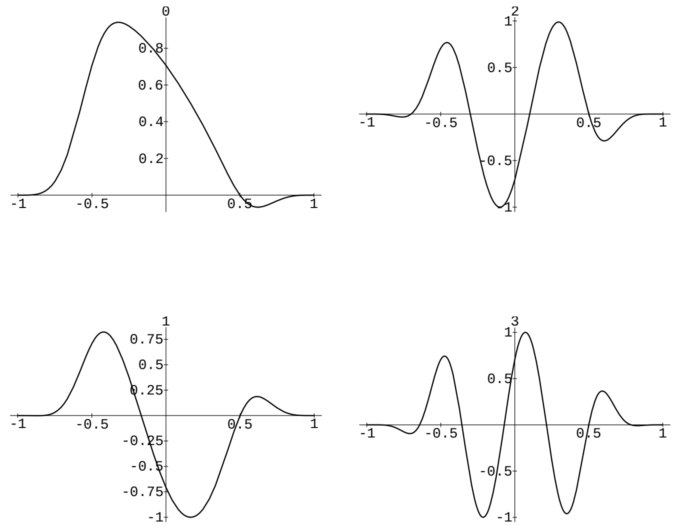
$$p_i^2(x) = 1 - p_i^2(2\alpha_{i+1} - x) \quad \text{for} \quad x \quad \text{near} \quad \alpha_{i+1}$$

then the functions

$$S_{i,k}(x) = \frac{2}{\sqrt{2\ell i}} p_i(x) \sin[(2k+1)\frac{\pi}{2\ell_i}(x - \alpha_i)]$$

form an orghonormal basis of $L^2(\mathbf{R})$ subordinate to the partition p_i. The collection of such bases forms a library of orthonormal bases. [3].

LCT



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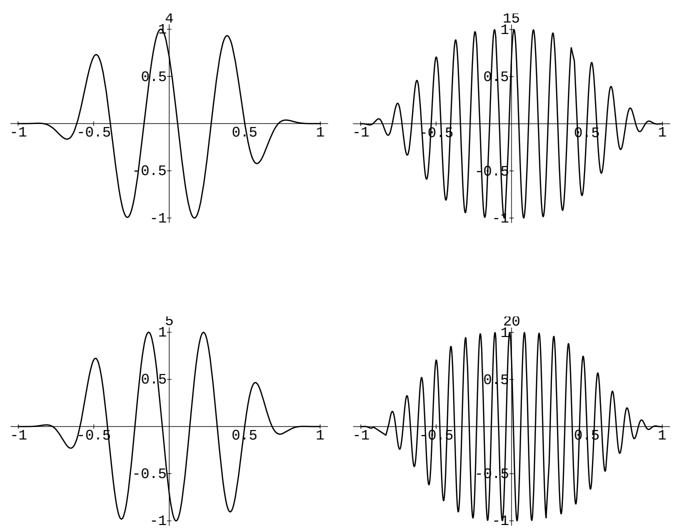


It is easy to check that if H_{I_i} denotes the space of functions spanned by $S_{i,k}$ $k =$

$0, 1, 2, \ldots$ then $H_{I_i} + H_{I_{i+1}}$ is spanned by the functions

$$P(x)\frac{1}{\sqrt{2(\ell_i + \ell_{i+1})}}\sin[(2k+1)\frac{\pi}{2(\ell_i + \ell_{i+1})}(x - \alpha_i)]$$

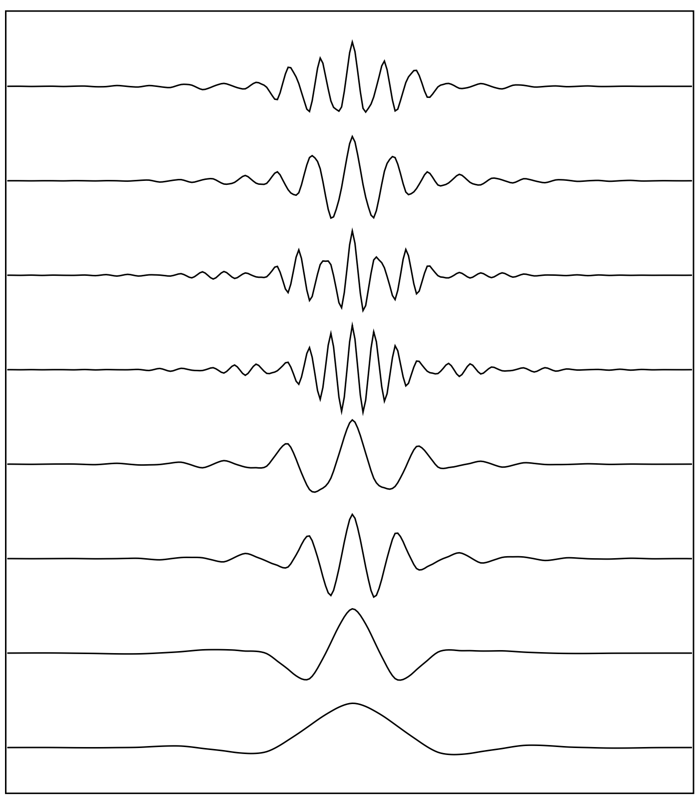
where

$$P^2 = p_i^2(x) + p_{i+1}^2(x)$$

is a "window" function covering the interval $I_i \cup I_{i+1}$.

Another new library of orthonormal bases called the Wavelet packet library can be constructed. This collection of modulated wave forms, corresponds roughly to a covering of "frequency" space. This library contains the wavelet basis, Walsh functions, and smooth versions of Walsh functions called wavelet packets.[1]

Wavelet-packets

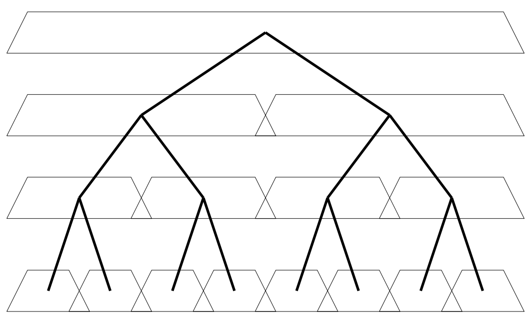


These waveforms are mutually orthogonal, moreover, each of them is orthogonal to all of its integer translates and dyadic rescaled versions. The full collection of these wavelet packets (including translates and rescaled versions) provides us with a library of "templates" or "notes" which are matched "efficiently" to signals for analysis and synthesis.[1],[4], Wavelet packet expansions correspond algorithmically to subband coding schemes and are numerically as fast as the FFT.

We were led to measure the "distance" or good fit between a basis and a function in terms of the Shannon entropy of the expansion.

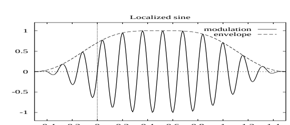
For example in the LST Library case

Tree search In the LST library



b(x) sin[(k+1/2)(x-a)/L]

a - endpoint ,
L -length

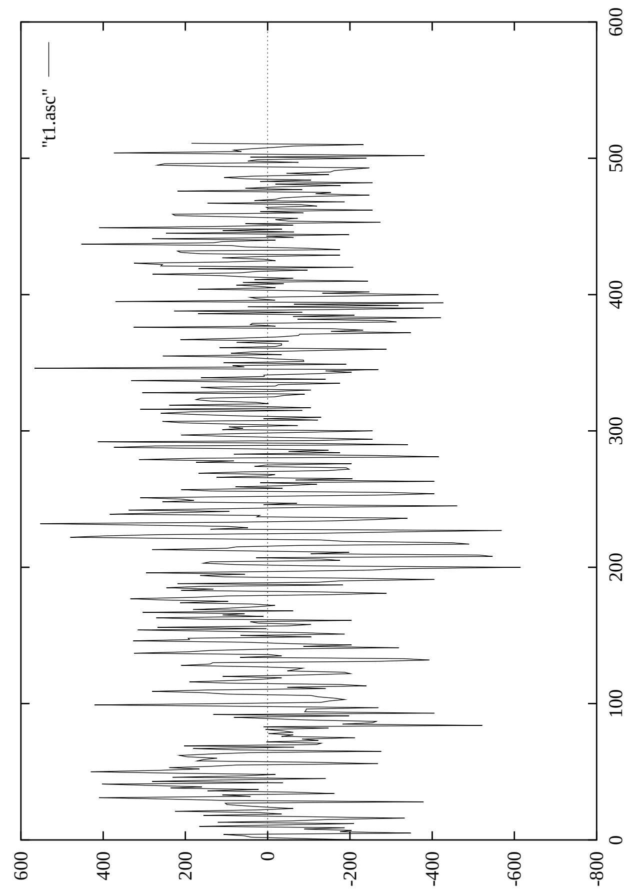


We compare the entropy of the expansion in two adjacent windows to the entropy of the expansion on their union.and pick the least expensive ,continuing the comparison with the selection made for the next pair ,etc. Ending up with the so called best basis of minimal cost . The wavelet packet best basis algorithm is similar but optimizes the segmentation in frequency space. This straighforward procedure is analoguous to a direct simple musical transcription.

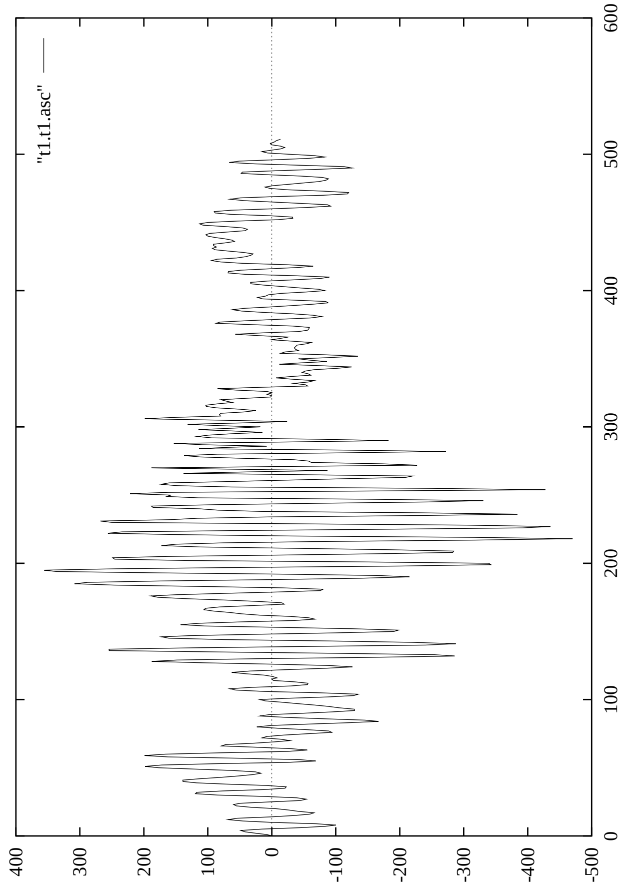
In order to extract structures out of a signal (orchestrate it) we will combine several expansions in different orthogonal functions out of distinct libraries of waveforms .We can view each library as a musical synthetiser (or instrument). Our task is to match the instruments to the signal . Another analogy explaining the idea behind these algorithms ,involves interpretation of sound in several languages ,say, English,French,Japanese. To an English dictionary Japanese is pure "noise" ,therefore the natural approach will be to correlate the sound to words in each dicationary separately pulling out the best correlations from each . Our approach here is to pick a best basis representation of the signal as long as it is well compressed (or well matched), the moment the compression rate deteriorates we stop and repeat the process on the residual.

This procedure is illustrated in the following figures where the original underwater noisy signal is peeled into layers as described above. We think of the well compressed part as coherent ,and of the residual as noisy.

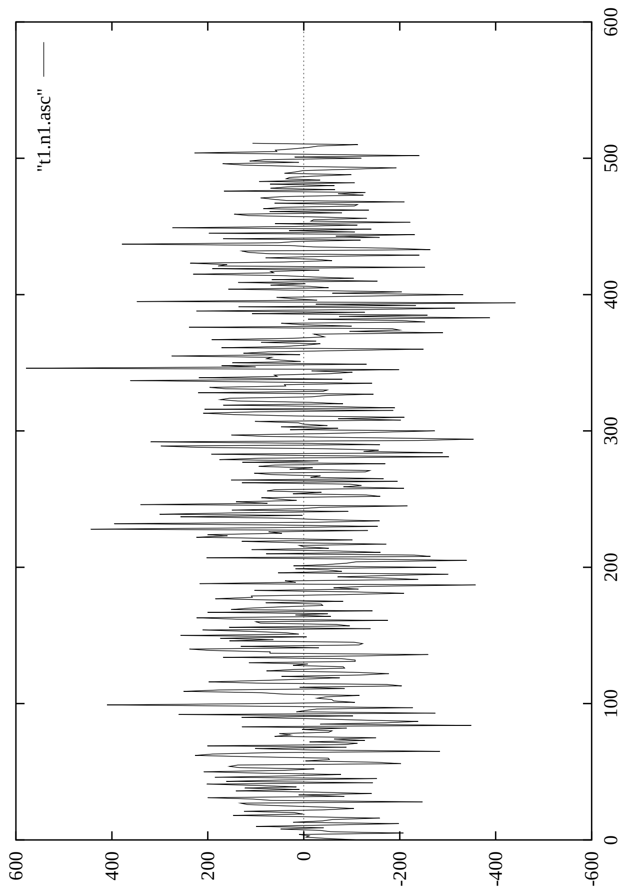
An underwater sound signal



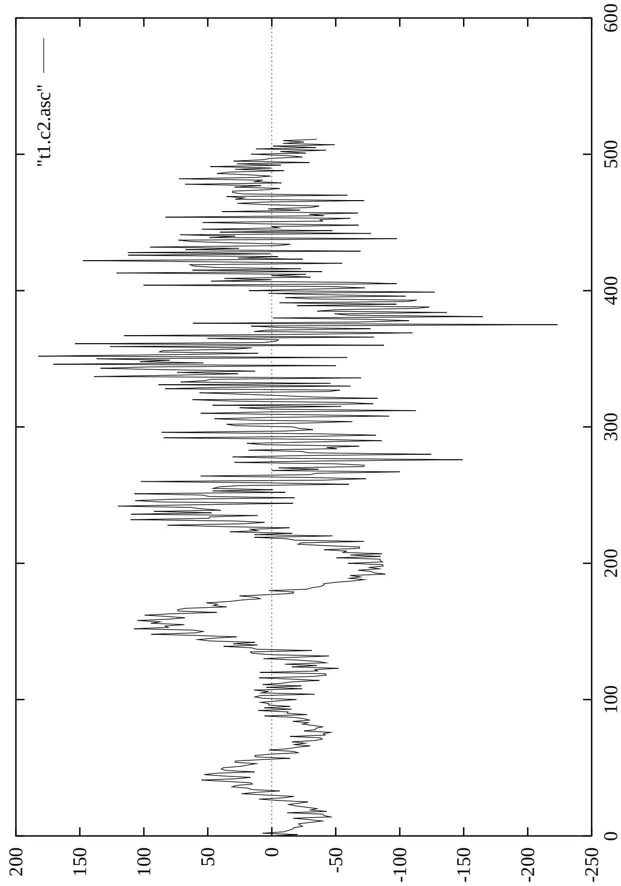
The first coherent component of the underwater sound signal,rough engine sound



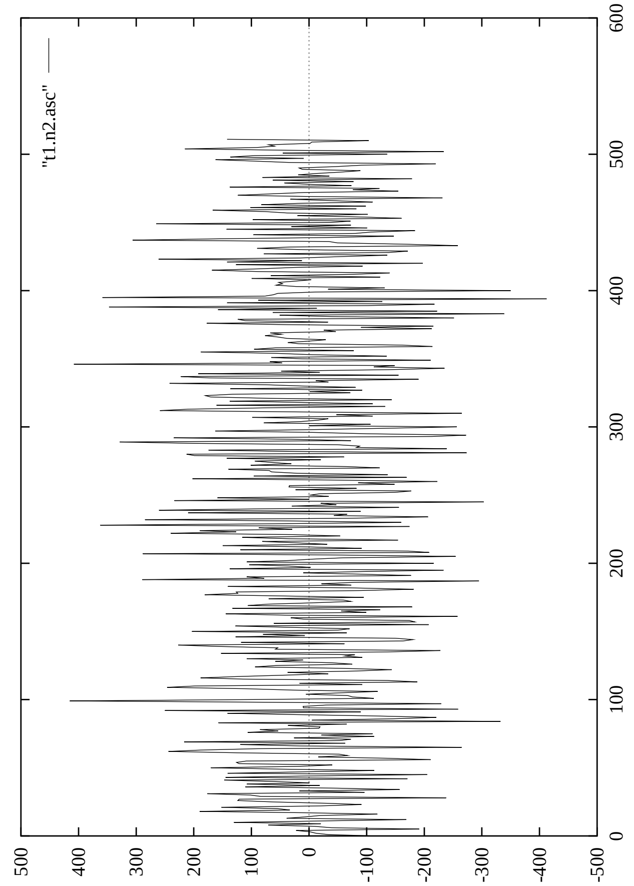
The first noisy residue of the underwater sound signal



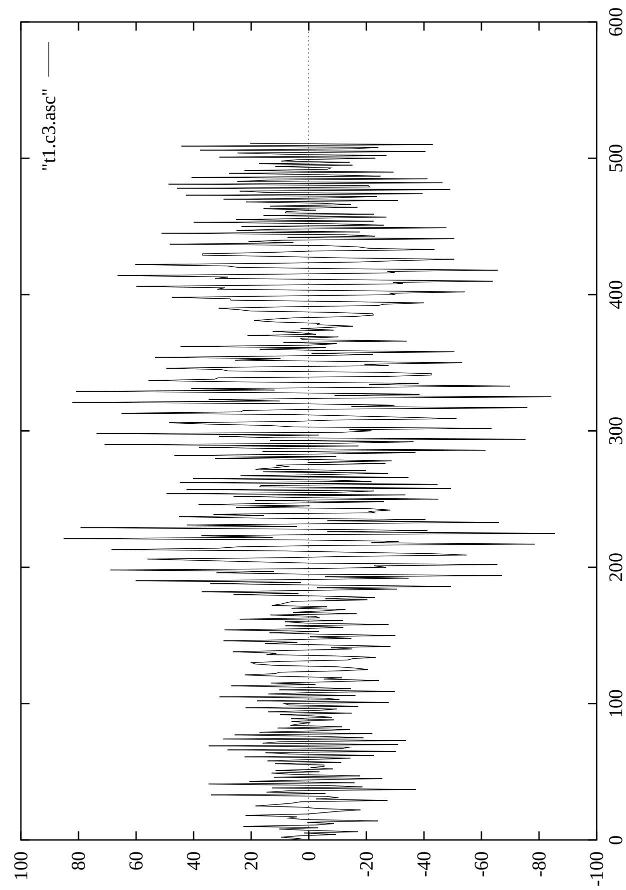
The coherent part of the preceding noisy part,perhaps a propellor sound



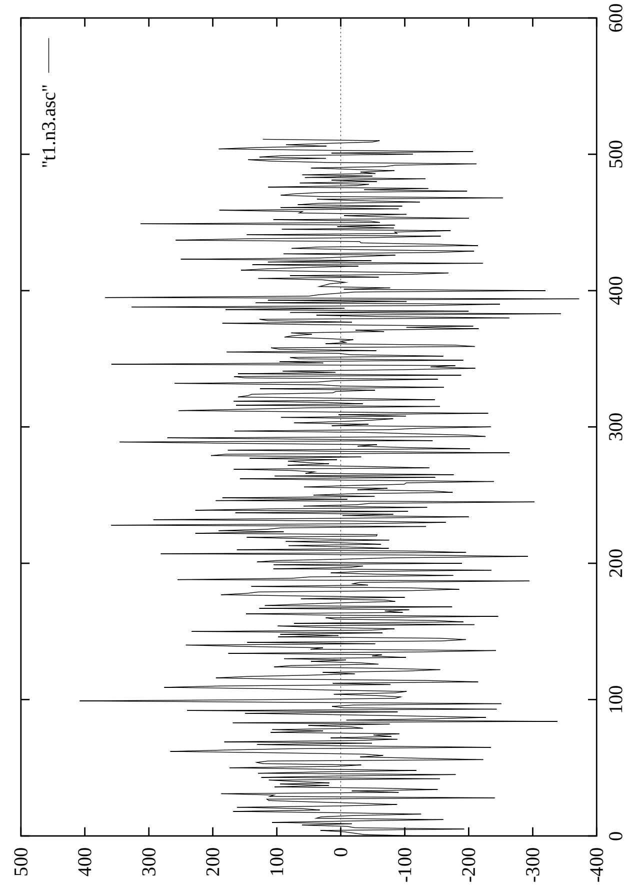
The second noisy component



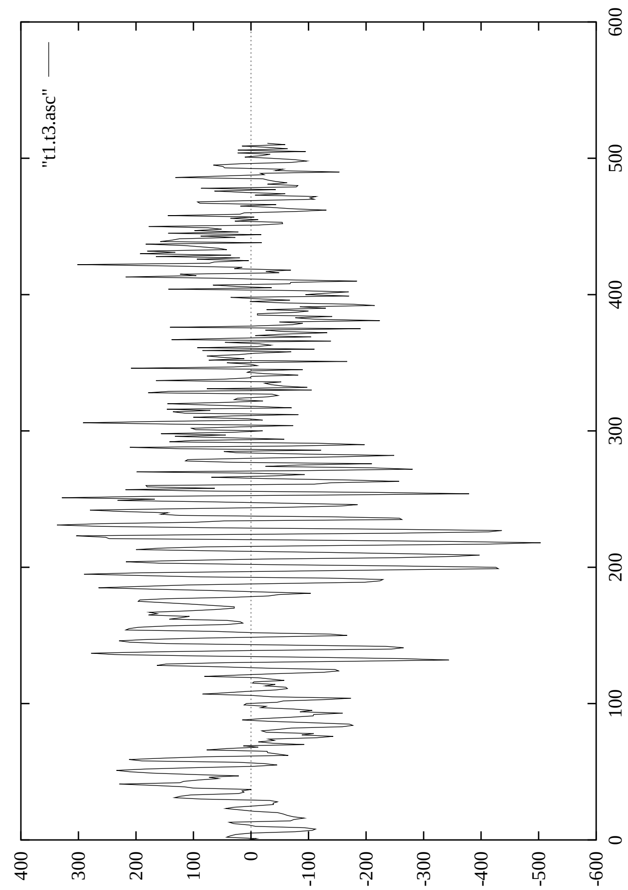
The third coherent component



The third noisy component



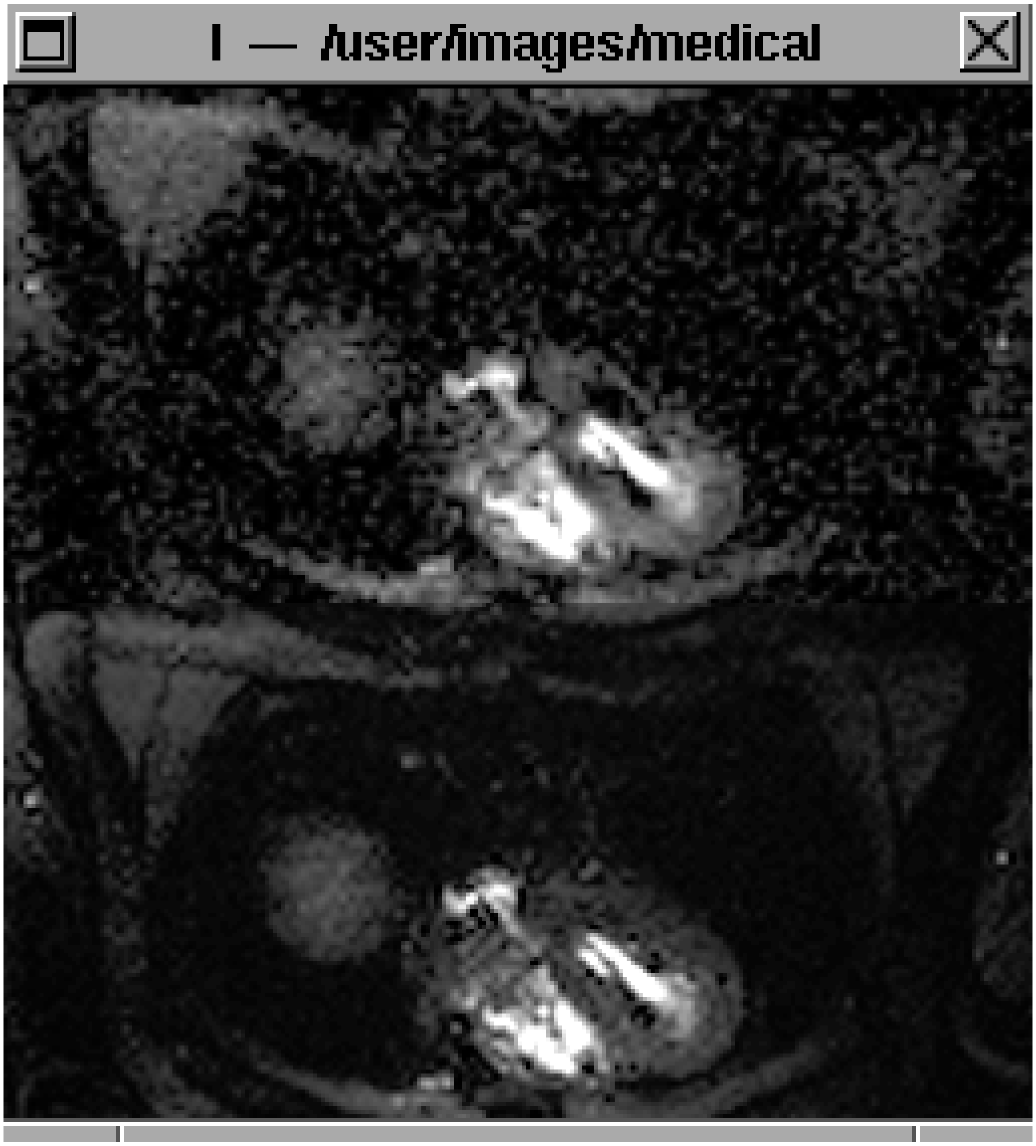
The sum of the coherent parts ," denoised signal"



Preprocessing.

As was discussed, variations on the denoising described in the above method have been tested as a tool for separation of clutter in SAR images as well as for elimination of " noise " in echoplanar MRI video images.(where the transcription algorithms provides a rudimentary model of the structure)

Echoplanar MRI noisy on top denoised as a video below



Typically this procedure of decomposing a signal into different layers adjusted by appropriate thresholds leads to an extraction of the "interesting" core part of the signal. We can use this core part as "clean" data for numerical compression and analysis. More specifically, numerical compression, in which a specific efficient wave form library is selected for representation of the data, results in a reduction of the number of parameters used for further processing algorithms .

This step is crucial for reducing the computation time on two levels: First, the expansion in the chosen waveform basis is fast (at worst of order $N \log N$, N is the number of samples). Second, the compression achieved reduces the dimensionaltiy of the problem. As an example, we point out that this procedure can substantially increase the speed of computation of the Karhunen Loeve basis and corresponding expansion for statistical factor analysis. Similarly, the reduced set of coordinates can be the input to neural nets and other clustering and classification methods accelerating the computation time substantially . .

The possibility of selecting a best basis for compression is but one aspect of a class of waveform selection algorithms. We developed various methods for selection of best bases for discrimination among different classes of signals as well as for classification. Such bases can be found by choosing cost function for waveform selection for which a waveform has low cost if it correlates well with targets in a desired class and correlates weakly with the other classes. We are currently experimenting with a variety of such cost functions (as defined by the classifier) .

We feel that these need to be constructed differently according to the nature of the target and the model at hand.

The obvious advantage of such methods is the speed of search for such bases. The tree search described previously eliminates the usual combinatoric explosion involved in these problems. Moreover such search actually provides discrimination features which should facilitate modeling. The procedure here is very similar to the methods used by statitisticians when using Classification and Regression Trees (CART) .

In conclusion, while it is possible that such preprocessing methods might be incorporated directly into diagnostic and ATR systems, it is more likely that these methods will serve more as an analysis tool to enable the modeling and construction of specific processors.

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