Abstract

The continuance of a rise in prices in a Western economy well after a downturn in final demands, termed "Stagflation," has been a puzzle to economists in the 20th century: we hope, from the results set out in this paper, that any stagflation encountered in the 21st century will have been understood and even anticipated (in the proper sense of that word) by appropriate economic policies which include the use of input-output (or interindustry) tables. In conclusion, at the end of this paper, attention is drawn to the computability of projections of industrial price-levels and rates of return: the "duals" to the familiar "primal" projections (embracing industrial outputs and growth rates) into the future. This leads one to a conjoint "forward view" (see Gielnik, 1980, in *Input, Output, and Marketing*, London, I.-O. P. C.) which should be applicable at least to most economies which have enough of the required data.

Keywords: Stagflation,Trade Cycle, Input-Output

Prices, the Trade Cycle, and the Nature of Industrial Interdependence

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1 Introduction

The rearrangement of the flows between and within industries—set out in dollars—in the American economy can be done through row and column interchanges or by other methods so as to clarify the economy's industrial structure. When that is done, certain implications of disturbances in industrial prices can be considered. This exercise has been done for the years for which input-output tables were available in the 20^{th} century; in particular 1919, 1929, 1939, and 1947. After 1947 we have all the tables built by the U.S. Department of Commerce for 1958, 1963, and at intervals thereafter, and recently annually. Most of the calculated results have been published previously, *e.g.*, (Evans and Hoffenberg 1952) and (Yan and Ames 1965), but we study implications different from theirs; also we fill in a few gaps.

2 Definitions and Structures

All the above tables are square matrices, having as many rows as there are columns respectively for industries' sales and purchases, and all entries are non-negative.

(i) A *block lower triangular matrix* with n industries has square submatrices not necessarily the same size arranged down the main diagonal. The simple example which we use below can be pictured as follows:

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{B} \end{pmatrix},$$

where $\mathbf{A}$ has r rows and columns, $\mathbf{B}$ has $n-r$ rows and columns, $\mathbf{C}$ has $n-r$ rows and r columns, and $\mathbf{0}$ has r rows and $n-r$ columns. Also, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ have non-negative entries while $\mathbf{0}$ has only zeros as entries. The whole matrix $\mathbf{M}$ is termed *reducible-indecomposable*.

(ii) An *irreducible* matrix can be set out as below:

$$\mathbf{H} = \begin{pmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{G} & \mathbf{F} \end{pmatrix},$$

where $\mathbf{E}$ has strictly fewer rows r than columns s and $\mathbf{F}$ has strictly fewer columns $n-s$ than rows $n-r$, and both $\mathbf{E}$ and $\mathbf{F}$ are *non-square*. $\mathbf{E}$, $\mathbf{F}$, and $\mathbf{G}$ have non-negative entries, but $\mathbf{0}$ has only zero entries.

(iii) Other kinds of square matrices exist, such as the ones in (Blakley and Gossling 1967) and (Gossling 1972), but these are not relevant to this paper.

3 Systems of Equations

The entries in $\mathbf{M}$ and $\mathbf{H}$ can be scaled to unit levels of industries' operations, and if considered as so scaled as $\grave{\mathbf{M}}$, $\grave{\mathbf{H}}$, then they yield systems of equations such as:

$$\begin{aligned} [\mathbf{I} - \grave{\mathbf{M}}]\mathbf{x} &= \mathbf{y} \\ [\mathbf{I} - \grave{\mathbf{H}}]\mathbf{x} &= \mathbf{y} \end{aligned}$$

where $\mathbf{x}$ is the vector of industries' levels of operation, and $\mathbf{y}$ the vector of final demands, and:

$$\begin{aligned} [\mathbf{I} - \grave{\mathbf{M}}^T]\mathbf{p} &= \mathbf{v} \\ [\mathbf{I} - \grave{\mathbf{H}}^T]\mathbf{p} &= \mathbf{v} \end{aligned}$$

where $\mathbf{p}$ is the vector of industries' price levels and $\mathbf{v}$ is the vector of values added by industries at unit levels of operation. (Note that $\grave{\mathbf{M}}^T$ and $\grave{\mathbf{H}}^T$ are the respective transposes of $\grave{\mathbf{M}}$ and $\grave{\mathbf{H}}$; *e.g.*, to get $\grave{\mathbf{M}}^T$ write the rows of $\grave{\mathbf{M}}$ as columns. $\mathbf{I}$ is the identity matrix with ones on its main diagonal and zeros everywhere else.)

In solving equations involving reducible-indecomposable matrices $\mathbf{M}$, one can see that equations for the first r rows of $[\mathbf{I} - \grave{\mathbf{M}}]$ can be solved on their own, and the results can be substituted into and used to solve the equations for the remaining $n - r$ rows of $[\mathbf{I} - `\mathbf{M}]$: a two-stage process. In contrast, this is impossible for solving systems involving irreducible matrices $\mathbf{H}$; the system has to be solved in one entire operation.

These solutions of systems bring out a special property of the positive entries of $\mathbf{H}$: that there is at least one cyclic chain of no less than n distinct entries such as $h_{ij}, h_{jk}, h_{kl}, \ldots, h_{ni}$; $\mathbf{H}$ is a *cyclic* matrix. In contradistinction, $\mathbf{M}$ is *non-cyclic*: one cannot find a complete cyclic chain of n entries amongst the positive entries of $\mathbf{M}$. A set of industries with an $\mathbf{H}$ structure implies that the industries are all each others' servants: this is *not* the case with an $\mathbf{M}$ structure.

4 Empirical Procedures

Inspection of input-output flow matrices usually leaves one puzzled, because they are not neatly set out like $\mathbf{M}$ and $\mathbf{H}$ in §2 above. Our central problem is to find out whether the flow matrices are *cyclic* or *non-cyclic*. In reverse historical order we have:

(a) Our Markov chain method (Wickerhauser 2003) used in this paper.

(b) The power series approach—implicit (Yan and Ames 1965);

(c) Gossling's $[\mathbf{I} + \mathbf{P}]^T$ and $\mathbf{C}$ matrices' columns of zeros (Gossling 1964);

(d) Matrix theory results: (Gantmacher 1959): a nonnegative small square cyclic matrix $\mathbf{A}$ has all entries in $[\mathbf{I} - \mathbf{A}]^{-1}$ positive;

(e) Rearrangement of rows and columns, by hand or with computer assistance, to arrive at $\mathbf{M}$ or $\mathbf{H}$ in the result.

Methods (a), (b), and (c) were used on the (Leontief 1960) *Structure of American Economy* tables for 1919, 1929, and 1939, and (d) was used for (Evans and Hoffenberg 1952) table for 1947; (d) can also be used on all later U.S. Department of Commerce tables for the American Economy; (e) we have avoided because of the computational complexity that is required.

5 Results

For the 1919 table the "upper block" industries which sold only within and amongst themselves and to final demand were those in Table 1.

Table 1. Upper block in 1919

No.	TITLE
3.	Canning and Preserving
4.	Bread and Bakery Products
6.	Liquor and Beverages
7.	Tobacco Manufacturers
9.	Butter, Cheese, etc.
10.	Other Food Industries
15.	Automobiles
33.	Clothing
36.	Leather Shoes
42.	Services

For the 1929 table the list of industries is the same, except that No. 9, Butter, Cheese, etc., is not included because that industry sold to Paper and Wood Pulp (Industry No. 29); Nos. 9 and 29 thus belong to the "lower block" set of industries.

For the 1939 table the list is reduced to just one industry: Eating Places—which is part of "Services" in the two previous tables. Also, the submatrix of the remaining 1939 industries is irreducible, *i.e.*, cyclic.

Inspection of the 1947 tableaux published in Evans and Hoffenberg (1952) brings one to their "Table 6—Direct and Indirect Requirements per dollar of Final Demand, 1947," in which all entries in its 44 rows and columns are strictly positive. In the notation of §3 of this paper one has $[\mathbf{I} - \grave{\mathbf{H}}]^{-1}$ as a strictly positive matrix and thus $\grave{\mathbf{H}}$ (and $\mathbf{H}$, the unscaled flows matrix) are irreducible and cyclic.

Beyond 1947, the reader may refer to the "direct and indirect requirements" tables for 1958, 1963, and years thereafter, as published by the U.S. Department of Commerce. One ventures the sweeping statement[1] that all these tables are strictly positive, as for 1947. *I.e.*, beyond World War II, "irreducibility rules."

[1]See, for example, the line on or in *The New Yorker*, attributed to J. Thurber: "There are no pianos in Japan."

6 Implications

For the first three tables, 1919, 1929, and 1939, one can see that an "upper block" industry can directly and indirectly make purchases from the "lower block" industries, but there is no back connection between the lower block industries and the upper block industries. Thus in these cases, a rise in final demand for an upper block industry's output is eventually constrained, as its level of output is raised, by industrial price rises in the *lower* block industries. As the latter are not sold *to* by the upper block industries, there is no way that the *upper* block industries can pass on industrial-purchase price increases to the lower block industries. We are once more ensconced in an Olde Trade Cycle World: Copper's up!

For 1947 and after, a price rise originating in any particular industry gets passed on to every other industry and back to that particular industry. Both demand inflation and cost inflation hits a supplier of this particular industry. A substantial price rise will continue to circulate for more than one such round, *i.e.*, for a few to several rounds as, *e.g.*, "OPEC I" and "OPEC II." This implies a considerably lengthy die-away in an initial industrial price rise; it will *even continue after industrial demand has slackened off from a previous, higher level.*

We come to the *Economic Journal* article (Wiles 1973), luckily published before "OPEC I" (!), where that author is implicitly assuming the inter-war 1919–29–39 industrial structure — particularly at the end of §9 and the beginning of §10 in his paper (pp.385–386 in (Wiles 1973)). There, an increase in a final commodity's demand goes down a supply chain and impinges on a "primary" (*i.e.*, lower block industry) commodity, whose price rises, and this raises prices up the supply chain to the "final buyer." This analysis is incomplete for the November 1957 situation with which Wiles' paper starts (p.377 of (Wiles 1973)). It would be sardonic to say one could get only so far without input-output tables; the New York Times, with respect, missed a very big "scoop." Since irreducibility rules *after* World War II we have the "New" Trade Cycle distinguished by Stagflation—where prices continue to rise long after an industrial recession has begun. Appropriate Trade Cycle policies now require prior verification of interindustrial interdependence.

7 Envoi: Prices in the 21st Century

In an early edition of (Sampson 1965) *Anatomy of Britain Today* (p.272) there is that nice quotation: "Any fool can find the answers to the questions; the difficult thing is to find the questions to the answers." We are now truly spoilt by the availability, in the U.S., the U.K., and elsewhere, of national input-output tables produced currently and annually. World-wide tables, complicated by fluctuating foreign exchange rates, have been produced as well, so a global picture of structural technological change over time emerges. However, in all the discussion of various price indices, indicating deflationary or inflationary trends, which is currently fashionable, there is no mention of future prices. We have present and future growth rates of industrial outputs, but future prices of industry outputs discounted from the present (see (Gossling and Sanchez-Garcia 1996)) remain a very academic concept; this should not be so.

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Appendix I Sources, Data and Computational Considerations

The 1919, 1929, and 1939 input-output flow tables used in this study were taken from Appendix C of *Productivity Trends* (Gossling 1972), which see. If anything, these are slightly more dense, flow-wise, compared to their corresponding originals: the three inter-war tables that can be found as fold-outs in the third edition of *Structure of American Economy* (Leontief 1960) and that are usually referred to—and their industrial classification denoted by—that title.

The *Productivity Trends* 1919 and 1929 tables have a 42-industry classification: the last, added industry being "Services." it is not present in the (Yan and Ames 1965) study for both these years; their study used the *Structure of American Economy* (Leontief 1960) original tables for 1919 and 1929. For the 1938 *Productivity Trends* table there are 38 industries, the last three being Nos. 36, 38, and 40 in the *Structure of American Economy* classification, respectively "Trade," "Business and Personal Services," and "Eating Places." Sectors 37, "Foreign Trade," and 39, "Government and Households," are omitted.

All the above three tables have physical capital investment flows included in industrial sales and purchases. For the 1947 table these flows are all assigned to final demand, which makes the result—a strictly positive tableau of current direct and indirect requirements—all the stronger.

As discussed in Appendix II below, the three inter-war tables indicate positive or apparently absent flows between and within industries. In our analysis the *Productivity Trends*' Appendix C Tables 1, 2, and 3 (Gossling 1972) were each re-worked by putting a "1" wherever there was a positive flow, and a "0" wherever there was no flow.

It is not necessary to assume constant returns to scale when the $\mathbf{H}$ matrix in §2 is scaled to $\grave{\mathbf{H}}$, a small square non-negative matrix, so that $[\mathbf{I} - \grave{\mathbf{H}}]$ can be inverted: the scaling can merely be seen as a mathematical operation. $[\mathbf{I} - \grave{\mathbf{H}}]$ can be inverted in a number of ways. In particular, there is the power series inversion where

$$[\mathbf{I} - \grave{\mathbf{H}}]^{-1} = \mathbf{I} + \grave{\mathbf{H}} + \grave{\mathbf{H}}^2 + \grave{\mathbf{H}}^3 + \cdots = \sum_{r=0}^{\infty} \grave{\mathbf{H}}^r. \tag{1}$$

This is the series of powers used in the study (Yan and Ames 1965).

In the article (Evans and Hoffenberg 1952), for their Table 6 of "direct and indirect requirements," there is a general review of the methods used to obtain this Leontief inverse matrix including power series and Gauss-Seidel iteration.

Appendix II Markov Chain Method

A square $n \times n$ matrix $\mathbf{Z}$ containing only ones and zeros defines a flow graph on n industries. A "1" at position (i, j), or row i, column j, represents a flow from industry i to industry j; a "0" at position (i, j) indicates no flow. Denote this number by $\mathbf{Z}(i, j)$. The matrix power $\mathbf{Z}^r$ has the number of distinct flow paths of length r at position (i, j), *i.e.*, those taking r steps to get from industry i to industry j. Thus $\mathbf{Z}^r(i, j) = 0$ for all $r = 1, 2, \ldots$ if there is no flow at all from industry i to industry j; otherwise, $\mathbf{Z}^r(i, j) > 0$ for some r.

Define the *connection matrix* $\mathbf{C}$ derived from $\mathbf{Z}$ to be the $n \times n$ matrix that has a "1" at position (i, j) if there is a flow path of any length from industry j to industry i. Thus $\mathbf{C}(i, j) = 1$ if and only if $\mathbf{Z}^r(i, j) > 0$ for some $r = 1, 2, 3, \ldots$. Using Equation 1 of Appendix I and the fact that $\mathbf{Z}^r(i, j) \geq 0$ so that there is no cancellation, it is easy to see that $\mathbf{C}(i, j) = 1$ if and only if $[\mathbf{I} - \mathbf{Z}]^{-1}(i, j) > 0$. It costs about n^3 arithmetic operations to compute this inverse.

However, $\mathbf{Z}$ needs to be small enough so that its "matrix norm" $\|\mathbf{Z}\|$ is less than 1, or else the infinite sum will not be convergent and the formula will fail. This problem is most often solved by choosing a constant c with $0 < c < 1/\|\mathbf{Z}\|$ and normalizing $\mathbf{Z} \mapsto c\mathbf{Z}$, at the cost of a little more arithmetic to compute $\|\mathbf{Z}\|$. However, for some c the positive numbers in $[\mathbf{I} - c\mathbf{Z}]^{-1}$ can be very tiny, even comparable to the round-off error of the computer performing the calculations. In those cases it is difficult to decide whether a tiny $[\mathbf{I} - c\mathbf{Z}]^{-1}(i, j) > 0$ truly indicates $\mathbf{C}(i, j) = 1$.

To avoid this problem, one can use integer arithmetic and sum the powers of $\mathbf{Z}$ directly. Since the longest possible cycle in an $n \times n$ flow matrix has n steps, it is only necessary to compute $\mathbf{S} = \sum_{r=0}^{n} \mathbf{Z}^r$. Then $\mathbf{C}(i, j) = 0$ if and only if $\mathbf{S}(i, j) = 0$. Computing $\mathbf{S}$ costs more arithmetic, n^4 operations, but this is not of practical concern for contemporary computers with $n < 200$.

However, it is not even necessary to compute matrix powers with integer arithmetic. Simpler "logical" arithmetic is sufficient, wherein to compute $\mathbf{PQ}$ for $n \times n$ matrices $\mathbf{P}$ and $\mathbf{Q}$ one uses the formula $\mathbf{PQ}(i, j) = 1$ if ($\mathbf{P}(i, 1) = 1$ and $\mathbf{Q}(1, j) = 1$) or ($\mathbf{P}(i, 2) = 1$ and $\mathbf{Q}(2, j) = 1$) or ... or ($\mathbf{P}(i, n) = 1$ and $\mathbf{Q}(n, j) = 1$). This may be denoted by the ordinary matrix multiplication formula,

$$\begin{aligned} \mathbf{PQ}(i, j) &= \sum_{k=1}^{n} \mathbf{P}(i, k) \times \mathbf{Q}(k, j) \\ &= [\mathbf{P}(i, 1) \times \mathbf{Q}(1, j)] + \cdots + [\mathbf{P}(i, n) \times \mathbf{Q}(n, j)], \end{aligned}$$

if $+$ is interpreted as "or" and $\times$ as "and." These logical operations are efficiently implemented in contemporary computers. If $n \leq 64$, for example, a 64-bit computer can evaluate the logical sum in just one operation rather than n, reducing the cost of this method back to n^3.

Once the connection matrix $\mathbf{C}$ is found, the eigenvector of its largest eigenvalue has a positive coordinate at every industry that is part of the maximal cycle. The eigenvalue itself is the number of vertices in that maximal cycle. Any linear algebra system may be used to compute the eigenvalues and eigenvectors.

Tableaux 1, 2, and 3 below present the results of our method applied to three rescaled input-output flow matrices. Across the top of each matrix and down the left side, labeling rows and columns, is the final digit of the industry index for that row and column. When the final digit would be 0 there is instead a space to aid readability.

A short program in the Standard C language, used to compute the connection matrices for the tables discussed in this article, is available from the second author's web site (Wickerhauser 2003). The input files describing the matrices in machine readable form together with the results printed as Tableaux 1, 2, and 3 are also posted at that site.

TABLEAU 1

```
ORIGINAL MATRIX Z1:                             EVENTUAL CONNECTION MATRIX C1:

+-123456789 123456789 123456789 123456789 12    +-123456789 123456789 123456789 123456789 12
1 111111111100000000000000010010010010000000    1 111111111111111111111111111111111111111110
2 100100000000000000000000000000000000000000    2 111111111111111111111111111111111111111110
3 000000000000000000000000000000000000000000    3 000000000000000000000000000000000000000000
4 000000000000000000000000000000000000000000    4 000000000000000000000000000000000000000000
5 101111001100000000000000000000010000000000    5 111111111111111111111111111111111111111110
6 000001000000000000000000000000000000000000    6 000001000000000000000000000000000000000000
7 000000000000000000000000000000000000000000    7 000000000000000000000000000000000000000000
8 100100011000000000000000010000010010000000    8 111111111111111111111111111111111111111110
9 000100000100000000000000000000000000000000    9 000100000100000000000000000000000000000000
  000100000100000000000000000000000000000000      000100000100000000000000000000000000000000
1 000000000001100000000000000000000000000000    1 111111111111111111111111111111111111111110
2 000000000000111000000000000000000000000100    2 111111111111111111111111111111111111111110
3 100000000011111100110100000000000000010110    3 111111111111111111111111111111111111111110
4 111111001101111100111100111110111111110110    4 111111111111111111111111111111111111111110
5 000000000000001000000000000000000000000000    5 000000000000001000000000000000000000000000
6 000000000000000110000000000000000000000000    6 111111111111111111111111111111111111111110
7 000000000000111011000000010000000000000100    7 111111111111111111111111111111111111111110
8 000000100100011000000000100000000000000100    8 111111111111111111111111111111111111111110
9 101011000001111010101000110010000010000110    9 111111111111111111111111111111111111111110
  000000000000101010111001010000000000001010      111111111111111111111111111111111111111110
1 111110111001111111110111111110110000011110    1 111111111111111111111111111111111111111110
2 011111111111111111101111111111111111011010    2 111111111111111111111111111111111111111110
3 000110000001111011101011010010010000001000    3 111111111111111111111111111111111111111110
4 000100000100111010111001110000100000001000    4 111111111111111111111111111111111111111110
5 011101111111111111111111111111111111111000    5 111111111111111111111111111111111111111110
6 100000010011001100101100010010110010010100    6 111111111111111111111111111111111111111110
7 100000000010011100001100111100000001001110    7 111111111111111111111111111111111111111110
8 000000100000001000000000000100000001000000    8 111111111111111111111111111111111111111110
9 000000000000000000000000000011100000000100    9 111111111111111111111111111111111111111110
  011111111111111111111111111111111111111110      111111111111111111111111111111111111111110
1 011111111111111111111111111111111111111110    1 111111111111111111111111111111111111111110
2 100000000000000100000000000010001110111100    2 111111111111111111111111111111111111111110
3 000000000000000000000000000000001000000000    3 000000000000000000000000000000001000000000
4 100000000000000000000000000100011000000000    4 111111111111111111111111111111111111111110
5 000000000000001000000000010100111011100000    5 111111111111111111111111111111111111111110
6 000000000000000000000000000000000001000000    6 000000000000000000000000000000000001000000
7 100000000000000000000000000000000000000000    7 111111111111111111111111111111111111111110
8 000000000000011000000000000000000000010000    8 111111111111111111111111111111111111111110
9 100000000000010000000000000100011000000100    9 111111111111111111111111111111111111111110
  011111011111111111101100111010110010010010      111111111111111111111111111111111111111110
1 110011010011110110101110011000000000000010    1 111111111111111111111111111111111111111110
2 000000000000000000000000000000000000000000    2 000000000000000000000000000000000000000000

Eigenvalues of C1:  (0,...,0,1,1,1,1,1,32)

Maximal cycle for tableau 1 (32 vertices):
(1,2,5,8,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,34,35,37,38,39,40,41)
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TABLEAU 2

```
ORIGINAL MATRIX Z2:                                 EVENTUAL CONNECTION MATRIX C2:

+-123456789 123456789 123456789 123456789 12        +-123456789 123456789 123456789 123456789 12
1 111111111110000000000000011110010010000000        1 111111111111111111111111111111111111111110
2 110100000000000000000000000000000000000000        2 111111111111111111111111111111111111111110
3 000000000000000000000000000000000000000000        3 000000000000000000000000000000000000000000
4 000000000000000000000000000000000000000000        4 000000000000000000000000000000000000000000
5 101111001100000000000000000000010000000000        5 111111111111111111111111111111111111111110
6 000001000100000000000000000000000000000000        6 000101000100000000000000000000000000000000
7 000000000000000000000000000000000000000000        7 000000000000000000000000000000000000000000
8 100100011100000000000000000001000001001000        8 111111111111111111111111111111111111111110
9 000100000100000000000000000010000000000000        9 111111111111111111111111111111111111111110
  000101000100000000000000000000000000000000          000101000100000000000000000000000000000000
1 000000000001000000000000000000000000000000        1 111111111111111111111111111111111111111110
2 000000000000111000000000000000000000000100        2 111111111111111111111111111111111111111110
3 100000000011111100110100000100000000001110        3 111111111111111111111111111111111111111110
4 111111111110111101111101111111111101111110        4 111111111111111111111111111111111111111110
5 000000000000001000000000000000000000000000        5 000000000000001000000000000000000000000000
6 000000000000000110000000001000000000000000        6 111111111111111111111111111111111111111110
7 000000000000111011000000001000010010001010 0        7 111111111111111111111111111111111111111110
8 000000100100111001000000110000100000001100        8 111111111111111111111111111111111111111110
9 101011000001111000100000110010000010011110        9 111111111111111111111111111111111111111110
  000110010100111011111001110010100000001000          111111111111111111111111111111111111111110
1 111111011100111111101011111010110000011110        1 111111111111111111111111111111111111111110
2 011111111111111111101111111111111111011010        2 111111111111111111111111111111111111111110
3 000110000001111011101011010000000000000000        3 111111111111111111111111111111111111111110
4 000100010101011001100001010000011100001000        4 111111111111111111111111111111111111111110
5 011101111111111111101111111111111111111010        5 111111111111111111111111111111111111111110
6 100001011110011101111100011010110010011100        6 111111111111111111111111111111111111111110
7 100000000010011100000100111110000011011110        7 111111111111111111111111111111111111111110
8 000000100000010000000000000000010001000000        8 111111111111111111111111111111111111111110
9 000000000000000000000000000000000111000000001100    9 111111111111111111111111111111111111111110
  011111111111111111111111111111111111111110          111111111111111111111111111111111111111110
1 011111111111111111111111111111111111111110        1 111111111111111111111111111111111111111110
2 100000000000011000000000001101111011111100        2 111111111111111111111111111111111111111110
3 000000000000000000000000000000001000000000        3 000000000000000000000000000000001000000000
4 000000000000000000000000000100111000100000        4 111111111111111111111111111111111111111110
5 000000000000001000000000010100101011101000        5 111111111111111111111111111111111111111110
6 000000000000000000000000000000000001000000        6 000000000000000000000000000000000001000000
7 100000000000000000000000000000000000100000        7 111111111111111111111111111111111111111110
8 100000000000011000000000000000000110010101 00        8 111111111111111111111111111111111111111110
9 100000000000011000000000000000000110010011 00        9 111111111111111111111111111111111111111110
  011111011111111111110110011101011001001001 0          111111111111111111111111111111111111111110
1 111011111111111111111110011111010110010000 00        1 111111111111111111111111111111111111111110
2 000000000000000000000000000000000000000000        2 000000000000000000000000000000000000000000
```

Eigenvalues of C2: (0,...,0, 1,1,1, 2, 33)

Maximal cycle for tableau 2 (33 vertices):
(1,2,5,8,9,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,34,35,37,38,39,40,41)

TABLEAU 3

ORIGINAL MATRIX Z3:

```
+-123456789 123456789 123456789 12345678
1 11000000000000000000011110110011000001
2 11000000000000000000010010101010000011
3 10111111011111110000000100000001001010
4 00101111011111110000000000000011001000
5 11001000000000000000000000000000100000
6 10000100000000000000000000000001000000
7 10001110110000000101010010000001101000
8 11000001000000000001100000000001101110
9 00000000100000000000000000000000100000
  10000000000000000000000000000000101000
1 11110001001001111111011111111101000100
2 00100111111111110000000000000000001000
3 00000000000000000000000000110000000110
4 00101001100001100001100000000001101110
5 11111111111111100111011100111011001111
6 11111111111111111111110001100011000000
7 11111010111111111000101100011110011111
8 11111111111111111111111111111111111110
9 11111110111111111111011111111111101000
  11111111111111111111011111111111101111
1 01000000001000101000111101110010000111
2 11111111110111111100111111111111001010
3 10001001000001000000001100000001000000
4 11000001011011100100110100101011000110
5 11100001001001111011111111111111101110
6 00000000000000000000000001000000001011
7 11001001001001111000011111111111000111
8 00000001000000000000010100010000000010
9 10000001001000000000010101111010000010
  11001101111011000100110011111111101110
1 10001001101011100000010010110011001111
2 11111111111111111111111111111110101111
3 11110000000000101110011000000000000000
4 11100000000100111101011111111110000000
5 11110111101111111111011110101100001100
6 11111111111111111111011111111110000100
7 11110111101111111111111111111111111111
8 00000000000000000000000000000000000000
```

EVENTUAL CONNECTION MATRIX C3:

```
+-123456789 123456789 123456789 12345678
1 11111111111111111111111111111111111111
2 11111111111111111111111111111111111111
3 11111111111111111111111111111111111111
4 11111111111111111111111111111111111111
5 11111111111111111111111111111111111111
6 11111111111111111111111111111111111111
7 11111111111111111111111111111111111111
8 11111111111111111111111111111111111111
9 11111111111111111111111111111111111111
  11111111111111111111111111111111111111
1 11111111111111111111111111111111111111
2 11111111111111111111111111111111111111
3 11111111111111111111111111111111111111
4 11111111111111111111111111111111111111
5 11111111111111111111111111111111111111
6 11111111111111111111111111111111111111
7 11111111111111111111111111111111111111
8 11111111111111111111111111111111111111
9 11111111111111111111111111111111111111
  11111111111111111111111111111111111111
1 11111111111111111111111111111111111111
2 11111111111111111111111111111111111111
3 11111111111111111111111111111111111111
4 11111111111111111111111111111111111111
5 11111111111111111111111111111111111111
6 11111111111111111111111111111111111111
7 11111111111111111111111111111111111111
8 11111111111111111111111111111111111111
9 11111111111111111111111111111111111111
  11111111111111111111111111111111111111
1 11111111111111111111111111111111111111
2 11111111111111111111111111111111111111
3 11111111111111111111111111111111111111
4 11111111111111111111111111111111111111
5 11111111111111111111111111111111111111
6 11111111111111111111111111111111111111
7 11111111111111111111111111111111111111
8 00000000000000000000000000000000000000
```

Eigenvalues of C3: (0,...,0, 37)

Maximal cycle for tableau 3 (37 vertices): all but vertex 38.