# *Wavelets: Tools for Science & Technology*, by Stéphane Jaffard, Yves Meyer, and Robert D. Ryan

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**Abstract**

Review of *Wavelets: Tools for Science & Technology*, by Stéphane Jaffard, Yves Meyer, and Robert D. Ryan, SIAM, Philadelphia, 2001, ISBN 0-89871-448-6

## 1 The New Version

This green book is an extensive revision of the blue SIAM paperback *Wavelets: Algorithms and Applications* [13], which itself was Robert Ryan's translation and revision of Yves Meyer's original pink book, *Ondelettes: Algorithmes et Applications* [12], which in turn evolved from Meyer's 1991 lectures to the Spanish Institute in Madrid. I reviewed the blue book [19] in 1994 and delighted in Meyer's succinct yet colorful perspective on the history and significance of wavelets. The new version, however, surveys a much larger territory. It is more than 100 pages longer than its predecessors. The only completely new parts are four appendices and a new Chapter 11 on lossy data compression and denoising, but there are substantial additions throughout.

The bibliography, for example, is three times bigger. At the end of 1993, the Wavelet Digest electronic mailing list completed its second volume with issue 19, sent to 3083 subscribers. This September, by contrast, issue 3 of volume 10 was sent to 18,916. This tremendous growth followed the incorporation of discrete wavelet transform algorithms into two image compression standards: 1993's WSQ [4, 6], and JPEG-2000 [7]. The preface of *Wavelets: Tools for Science & Technology* states that Chapter 11 was the authors' original reason to revise, but it is clear that a decade of research has changed the way the rest of the material has to be presented. There are 264 items in the new book's bibliography, plus an extensive author index, compared with 86 (an average of eight per chapter, with some repetitions), scattered about the earlier book. The new bibliography has unusual backward pointers, giving the page numbers where citations appear in the text, so that with a little work it is still possible to browse by chapter or subject.

Some chapters are strictly augmented with material that was not known in 1992. For example, orthonormal modulated Malvar wavelets, or "chirplets," appeared in 1996 and are in the new Chapter 6. A very important existence result for Navier–Stokes flows, which was discovered by Meyer's student Cannone in 1995, appears in the new Chapter 10. Wavelet transforms to repair Hubble Space Telescope images, which were superseded by a hardware fix in late 1993, became standard astronomical image enhancement tools by 1997 and are now discussed in a vastly expanded Chapter 12.

The four new appendices cover mathematical technicalities that, while important, would derail the narrative if discussed in the main text. For example Appendix A on quadrature filter fundamentals contains a complete proof that every continuous filter operator on finite-energy sequences is determined by the Fourier coefficients of some bounded periodic transfer function. None of the difficulties addressed in that proof arise in the finite impulse response (FIR) filter case, which includes all compactly-supported wavelets and most of the applications. Likewise, the complete details of a counterexample to Mallat's conjecture about perfect reconstruction from wavelet maxima are placed in Appendix C. The 1993 translation embedded a sketch of this counterexample in five pages of the chapter on human and computer vision, but the full story takes a dozen pages of calculations. The two other new appendices discuss details of the continuous wavelet transform inversion theory in $L^2$, and facts about Hölder and Besov spaces of functions.

Despite the new material, the same unifying ideas still bind the narrative. Meyer's original thesis was that the quantitative description of nature benefits from choosing appropriate analysis and synthesis tools, and that wavelets provide a huge and versatile toolbox. There is an effective meta-algorithm, or procedure for designers of wavelet algorithms, because of the remarkable mathematical properties of wavelets. Jaffard, Meyer, and Ryan discuss these properties in greater detail than Meyer alone did. His book was a brief catalog of the functions that had recently become available, and gives some of the formulas essential to using them in particular problems. The new version is more of an encyclopedia. The standard of mathematical rigor is high, and the amount of detail provided in the proofs is substantial. It may be that Meyer's earlier book sought to be equally definitive, but the results were simply not all there. Still, it is not a handbook or programmer's guide, but is rather an algorithm developer's guide, as not all details needed for implementations are presented. Some rather elementary results needed to implement the algorithms discussed in the book are omitted. Not mentioned at all, for example, is the symmetric extension discrete wavelet transform, which is used in both WSQ and JPEG-2000.

The standard of historical rigor is also quite high, although with so many contributors it is impossible to determine everybody's role beyond dispute. Some care was evidently taken to identify the first appearance of some ideas commonly attributed to the authors, such as smooth, localized, orthonormal wavelets. "Ideas rarely have well-defined beginnings," note the authors in Chapter 12, having said the same thing in other ways in preceding chapters as well. The ideas in question concern such problems as the large-scale structure of the universe, the information content of a signal, and the nature of turbulence. Only Chapter 2 follows a timeline to show how harmonic analysis since Fourier developed into wavelet analysis. In later chapters, the historical discussion becomes increasingly ideosyncratic. With the most challenging problems, where all known methods are unsatisfactory and therefore controversial, the first published wavelet application has sometimes been neither useful nor correct. It is a pleasure to observe how gracefully and respectfully the authors describe the pioneering work in these cases.

The new collaboration shines with other examples of Meyer's familiar colloquial style. Even in the totally new Chapter 11, the reader is asked rhetorically whether nonstationary signals are "a jungle, a terra incognita waiting for proper exploration." After this, there follows a series of theorems on rates of nonlinear approximation for some nonstationary signal classes, needed to judge their compressibility and to denoise them. The discussion closes with the admonition that "specific problems call for tailored solutions," a theme running through all the other chapters of this book.

## 2 The Competition

There are now many dozens, if not hundreds, of books on wavelets. Two that might be considered directly comparable are *Wavelets: A Mathematical Tool for Signal Analysis* by Charles K. Chui [1], based on his tutorial lectures at the 1993 SIAM Annual Meeting in Philadelphia, and the classic *Ten Lectures on Wavelets* by Ingrid Daubechies [2], based on her CBMS-NSF Regional Conference Lectures at the University of Lowell in 1992. Both of these older monographs lack the scope of *Wavelets: Tools for Science & Technology*, although they go into much greater detail about implementations of discrete wavelet transform algorithms.

Among the more mathematical wavelet books are several by Yves Meyer, starting with the first two volumes of *Ondlettes et Opérateurs* [10, 11], now translated into English. Those introduce wavelets as a construction which simplifies and automates a large body of theory in traditional "hard" analysis. There are also *A First Course in Wavelets* by Hernández and Weiss [3], *Analyse Continue par Ondelettes* by Torrésani [16], and *Fourier Analysis and Wavelets* by Kahane and Lemarié-Rieusset [9]. Again, the other books have narrower scope, but prove more theorems in their special areas.

More popular introductions include *The World According to Wavelets* by Hubbard [5], *Wavelet Analysis: The Scalable Structure of Information* by Resnikoff and Wells [14], and *Ripples in Mathematics: The Discrete Wavelet Transform* by Jensen and la Cour-Harbo [8]. It is difficult to explain wavelet analysis without some formal mathematics, but I think that if *Wavelets: Tools for Science & Technology* or any of these other books are read skipping formulas and proofs, they will be about equally informative.

Books that focus on algorithm implementations include *Wavelets and Filter Banks* by Strang and Nguyen [15], *Wavelets and Subband Coding* by Vetterli and Kovačević [17], and my own book, *Adapted Wavelet Analysis from Theory to Software* [18]. Some book like one of these is probably indispensible for programmers wishing to implement the algorithms in *Wavelets: Tools for Science & Technology*. In addition, the documentation that comes with software systems such as Matlab's Wavelet Toolbox, Mathematica's Wavelet Explorer, and S-Plus Wavelets sometimes has very good explanations of the underlying mathematics.

The many compilations of wavelet articles published after conferences and workshops are too numerous to mention. These can be useful, especially when focused on a particular application and read by experts in that application. But *Wavelets: Tools for Science & Technology* is a more cohererent introduction to the tool itself, and with its expanded bibliography is probably a better starting place for the nonexpert.

Jaffard, Meyer and Ryan's book stands out from its strong and numerous competitors because of its mathematical rigor, selectivity, and style. Not all possible applications are discussed, not all details of particular algorithms are investigated, not enough information is supplied to write a computer program. And yet, it gives succinct and elegant proofs of the main things we fully understand about wavelets, lists applications where wavelets have really worked, and charms the reader into an enlightened wariness with its graceful warnings and admonitions.

## References


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