# Ryan's translation of Meyer's *Wavelets : Algorithms and Applications*

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**Abstract**

Review of *Wavelets : Algorithms and Applications*, by Yves Meyer, Society for Industrial and Applied Mathematics, Philadelphia, 1993, softcover, pp. 133, $19.50, ISBN 0-89871-309-9

## 1 The wavelets and the applications

This is Yves Meyer's original pink book, *Ondelettes: Algorithmes et Applications* [7], but translated and revised by Robert D. Ryan. The books began as lectures delivered by Meyer to the Spanish Institute in Madrid in early 1992, and it presents a broad and somewhat opinionated picture of wavelet research and its outgrowths up to then. Ryan notes in the Translator's Foreword that he expanded on some of the statements in the French original and adjusted some notation for consistency, but was otherwise faithful to the text.

The book travels an enormous distance in a slim 133 pages. It describes the construction and the properties of various kinds of wavelets, including Grossmann–Morlet "continuous" wavelets, Daubechies "discrete" orthogonal wavelets, Gabor–Malvar wavelets, and libraries of wavelet packets and other time-frequency atoms. It places these objects in the context of many traditional and new algorithms: compression, analysis of signals in time and frequency, and feature detection through decomposition into basic units. At the end of the book there are four short chapters on vision, fractals, turbulence and cosmology, where some wavelet-like decompositions have recently yielded new results.

In effect Meyer develops the notion of a meta-algorithm based on choosing the appropriate decomposition units for a particular problem. His book is a brief encyclopedia of the functions that have recently become available, and gives some of the formulas essential to using them in particular problems. It is not a programmer's guide, but is rather an algorithm developer's guide, useful to the supervisor who must decide how to decompose in order to best reveal the features of interest or calculate the needed quantity.

Meyer writes in a fine colloquial style, which is preserved in Ryan's translation. There are references to Montaigne, Roland Barthes, and Mandelbrot's interview in *France-Culture*. In one chapter, Meyer writes how "with keen pleasure" he reread Galand's thesis introducing quadrature mirror filters in 1983. In another, he hopes that "readers who are mathematicians will enjoy reading this chapter as much as we have enjoyed writing it." The enjoyment that Meyer felt makes the text much more dynamic than, for example, a typical recent journal article on wavelets and applications.

The main charm of the text is that is uses mathematical rigor both to delineate where the new algorithms are valid, and also to indicate what we may expect if we go out of bounds. Thus, Meyer gives a counterexample in Chapter 8 showing that Mallat's reconstruction from edges does not always recover the exact image, but then explains "why Mallat's algorithm works in practice with such excellent precision": step functions are correctly recovered, and "the signals in question have more in common with step functions than with the subtle functions described in the counterexamples." In Chapter 7 he describes a "catastrophic phenomenon": Sére's result that Gabor wavelets can be arbitrarily separated in the idealized time-frequency plane yet still have enough overlap so that their superposition has infinite energy. Then he points out the root cause (arbitrary eccentricity of the time-frequency support rectangle) and describes various ways around the problem (Malvar wavelets and wavelet packets). He does not neglect to mention the advantages and disadvantages of either alternative, and includes a consideration of the computational costs involved.

## 2 The historical perspective

This chapter gives a succinct history of the representation of functions as superpositions of some kinds of elementary functions, with an emphasis on which properties can be dediced from the coefficients alone. There is a sketch of Fourier analysis and its problems, with the resulting mathematical outgrowths of Cesaro, Schauder, Lebesgue etc. It gives a quick construction of Brownian motion as sums of triangles, then sketches the Littlewood-Paley theory of Fourier series in order to show how sums of exponentials were the precursors of wavelets. There is a section devoted to Franklin's wavelets, Strómberg's wavelets, Lusin's wavelets and the atoms of Coifman and Weiss used to give real-variable characterizations of the Hardy spaces. The final section is devoted to developments from the Grossmann and Morlet school, including the continuous wavelet transform and the orthogonal or “fast” wavelet transform of Daubechies and Mallat.

Naturally, a 20-page summary with plenty of space devoted to the basic formulas and discoveries will omit many important contributions, especially from relatively recent times when the subject has grown most rapidly. One hopes that such omissions will not be taken too seriously. Even a superficial survey of the almost 1000 entries in Pittner, Schneid and Ueberhuber’s *Wavelet Literature Survey* [8] would require a second volume. Meyer instead presents those applications and contributions which in his judgment illustrate the prospects of wavelet analysis.

## 3 A comparison with some other books

Meyer has written several other books on wavelets, starting with the first two volumes of *Ondlettes et Opérateurs* [5, 6]; the third volume is joint work with R. Coifman. Those are much more mathematical, intended mainly to introduce wavelets as new machinery which simplifies and automates a large body of theory in traditional “hard” analysis.

There are now several good compilations of wavelet articles, including books like Ruskai’s *Wavelets and their Applications* [9] as well as special issues of journals like the March, 1992 *IEEE Transactions on Information Theory*. These provide an excellent survey of some of the applications so far explored, but naturally they cannot serve as a coherent source of information. *Wavelets: Algorithms and Applications* describes many of the works in the compilations and might even be used as an annotated bibliography prior to reading those other books.

Meyer’s new book makes a good introduction to wavelets and thus invites comparison with Chui’s *An Introduction to Wavelets* [2]. Chui mostly concentrates on the Battle–Lemarié spline wavelet construction, and leaves most of the applications to a much thicker second volume *Wavelets–A Tutorial in Theory and Applications*, [3], which is a compilation of many authors’ work. Likewise, Frazier and Benedetto’s compilation [1] contains numerous introductory articles on the applications of wavelet transforms.

Another related book is Daubechies’ *Ten Lectures on Wavelets* [4]. Daubechies also provides many relevant theorems needed to design good wavelet algorithms, but she delves much more deeply into the details of implementation. Her book is much longer than Meyer’s but has fewer applications, seemingly because they cannot be treated with the same rigor as the wavelet contruction itself. Most of the details which one might wish to know after reading Meyer's book, such as how to construct orthogonal wavelet filters, may can be found in Daubechies.

Meyer's book covers much ground and spends little time on implementation details, which paradoxically makes it quite useful for applications. This may be understood by an analogy. It is more generally useful to know the range of applicability of the wavelet transform than to know all the fine details of a particular wavelet, just as road maps are more interesting to a driver than the automobile's blueprints.

## References


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[2] Charles K. Chui. *An Introduction to Wavelets.* Academic Press, Boston, 1992. ISBN 0-12-174584-8.

[3] Charles K. Chui, editor. *Wavelets–A Tutorial in Theory and Applications.* Academic Press, Boston, 1992. ISBN 0-12-174590-2.

[4] Ingrid Daubechies. *Ten Lectures on Wavelets*, volume 61 of *CBMS-NSF Regional Conference Series in Applied Mathematics.* SIAM Press, Philadelphia, Pennsylvania, 1992. ISBN 0-89871-274-2.

[5] Yves Meyer. *Ondelettes et Opérateurs*, volume I: Ondelettes. Hermann, Paris, 1990. ISBN 2-7056-6125-7.

[6] Yves Meyer. *Ondelettes et Opérateurs*, volume II: Opérateurs. Hermann, Paris, 1990. ISBN 2-7056-6126-0.

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[8] Stefan Pittner, Josef Schneid, and Christoph W. Ueberhuber. Wavelet literature survey. Technical report, Institute for Applied and Numerical Mathematics, Technical University in Vienna, Austria, 1993. ISBN 3-901363-00-9.

[9] Mary Beth Ruskai et al., editors. *Wavelets and Their Applications.* Jones and Bartlett, Boston, 1992. ISBN 0-86720-225-4.