

Math 5022 - Complex Analysis 2
Homework 3

1. Prove that

$$\frac{1}{\Gamma(z)\Gamma(-z)} = -\frac{z}{\pi} \sin z.$$

2. Prove that

$$\Gamma(z)\Gamma\left(z + \frac{1}{2}\right) = 2^{1-2z} \sqrt{\pi} \Gamma(2z).$$

3. Calculate $\Gamma'(1)$.

4. Prove that

$$\frac{1}{z} \prod_{j=1}^{\infty} \left[\left(1 + \frac{1}{j}\right)^z \left(1 + \frac{z}{j}\right)^{-1} \right] = \lim_{n \rightarrow \infty} \frac{n! n^z}{z(z+1)(z+2) \cdots (z+n)}.$$

5. Prove that:

$$\gamma = - \int_0^{\infty} e^{-t} \log t dt = - \int_0^1 \log \log \frac{1}{t} dt.$$

6. Prove that:

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}.$$