Math 109 Homework 1, Fall 2022 (Answer Key)

1) a) E_5 b) $D^{\flat}_3$ c) $C^{\#}_6$ d) A_1

2) a) Perfect Fourth, 5 semitones
b) Perfect Fifth, 7 semitones
c) Perfect Fifth, 7 semitones
d) Minor Seventh, 10 semitones

3) a) $q = 3$, $r = 5$
b) $q = -4$, $r = 8$
c) $a = 3^{18} + 2$, $r = 1$
d) $q = 6k + 1$, $r = 2$

4) a) $\sim$ "lives in the same state as"
Reflexive: Yes, a person lives in the same state as themself
Symmetric: Yes; If person A lives in the same state as person B, then person B lives in the same state as person A.
Transitive: Yes, If person A lives in the same state as B AND B lives in the same state as C
Then person A lives in the same state as person C
This IS an equivalence relation
Elements are grouped by state of residence

b) $\mathbb{R}$; $\geq$
Let $a, b, c \in \mathbb{R}$
Reflexive: Yes, $a \geq a$ since $a = a$
Symmetric: No, For any a, b where $a \neq b$, if $a \geq b$, $b \not\geq a$.
Transitive: Yes, if $a \geq b$ and $b \geq c$, then $a \geq c$
This IS NOT a valid equivalence relation

c) $N \sim N' \iff$ the modular interval between N and N' is either the unison interval or a tritone (up or down)

Reflexive: Yes; a note is the unison interval with itself

Symmetric: Yes; the interval between two notes is the same upwards and downwards.

Transitive: Yes; Assume $X \sim Y$ and $Y \sim Z$
So, the interval from X to Z is a multiple of 6 semitones, and is a modular tritone or unison.

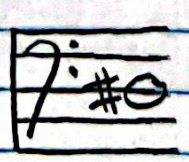
This IS a valid equivalence relation
Classes 6 semitones apart form each group

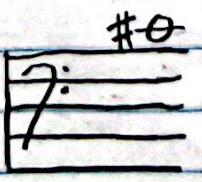
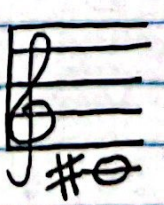
5) $k \equiv \ell \iff n \mid (k-\ell)$

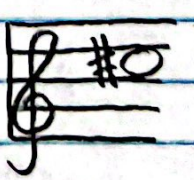
Reflexive: Yes, $k \equiv k$
$n \mid (k-k)$ which is the same as $n \mid 0$
For some fixed n, $n \cdot 0 = 0$

Symmetic: Yes; assume $k \equiv \ell$
so $n \mid (k-\ell)$ and $n \cdot q = k - \ell$
Consider $n \mid (\ell - k)$. $\ell - k = -(k-\ell)$
So, $n \cdot (-q) = -(k-\ell) = \ell - k$
Thus, $\ell \equiv k$

Transitive: Yes; assume $k \equiv \ell$, $\ell \equiv m$
So, $n \mid (k-\ell)$ and $n \mid (\ell - m)$
Consider $n \mid (k-\ell) + (\ell - m)$
$\Rightarrow n \mid (k-m)$
Thus $k \equiv m$

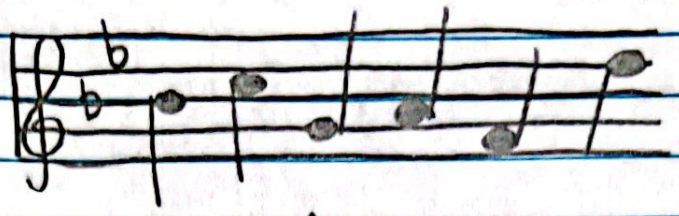
6) a)  $D^{\#}_3$

b)  OR  $C^{\#}_4$

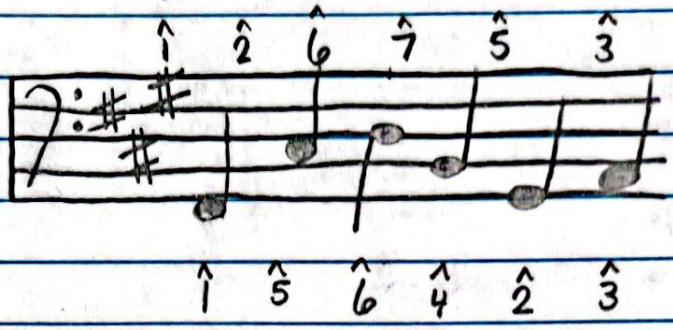
c)  $C^{\#}_5$

d)  D_4

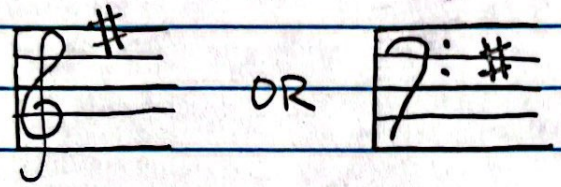
7) a)



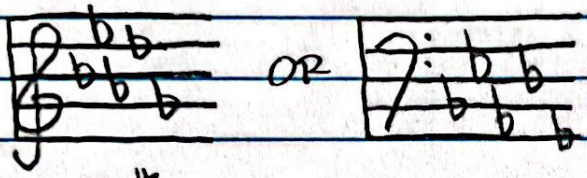
b)



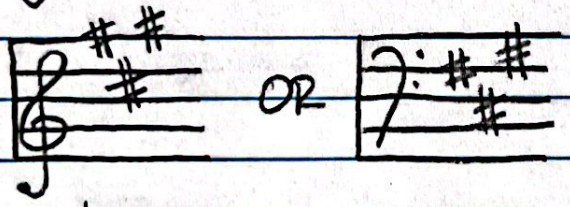
8) a)



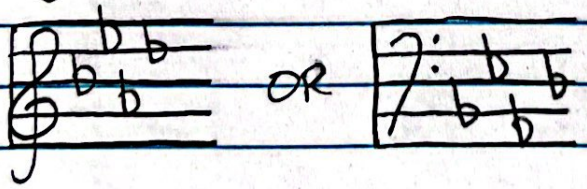
b)



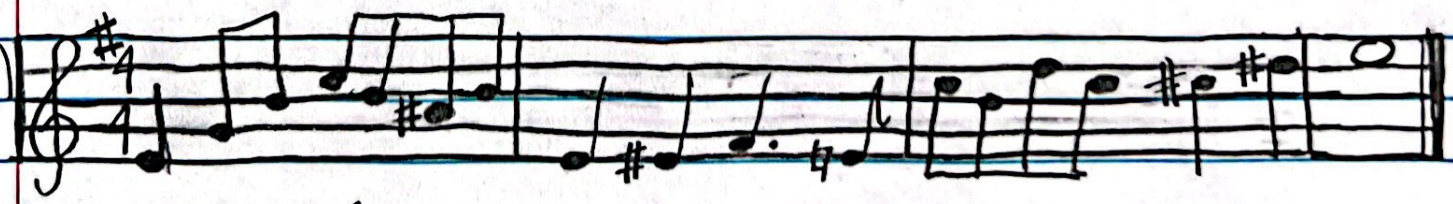
c)



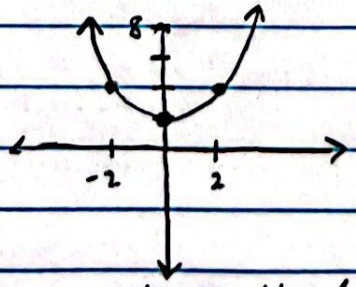
d)



9)

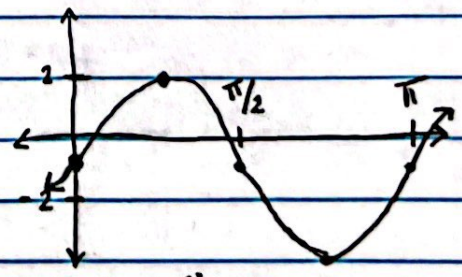


10) a)



- Vertical stretch by 1/2 (compression)
- shifted up by 2 (vertical shift)

b)



- Vertical shift down by 1
- Vertical stretch by 3
- Horizontal compression by 2