Math and Music HW #2 Solution

1. In 3/8 time:
   a) \( \frac{3}{16} \) of a beat
   b) \( \frac{21}{32} \) of a beat

In 12/8 time:
   c) \( \frac{1}{2} \) beats
   d) \( \frac{5}{12} \) of a beat

2. Let \( S = 1 + r + r^2 + \ldots + r^m \), \( r \in \mathbb{R}, m \in \mathbb{Z} \)

   \[ S \left(1 - r \right) = S - rS = 1 + r + r^2 + \ldots + r^m - \left(r + r^2 + \ldots + r^m + r^{m+1}\right) \]

   \[ = = \frac{1-r^{m+1}}{1-r} \]

   \[ \Rightarrow S(1-r) = 1 - r^{m+1} \]

   \[ r \neq 1 \Rightarrow S = \frac{1-r^{m+1}}{1-r} \quad \Box \]

This expression gives the duration of a dotted note where "1" represents the duration of the note undotted, \( r = \frac{1}{2} \), and \( m \) is the number of dots.

3. a) \[ \underbrace{\text{8th note}}_{\text{septuplet}} \]
   b) \[ \underbrace{\text{32nd note}}_{\text{quintuplet}} \]
   c) \[ \underbrace{\text{32nd note}}_{\text{non-octuplet}} \]

In 4/8 time:
   a) \[ \underbrace{\text{1 beat}}_{\text{quarter note}} \]
   b) \[ \underbrace{\text{2 beats}}_{\text{half note}} \]
4.

a) \( \frac{3}{4} \) | b) \( \frac{1}{2} \)

c) \( \frac{9}{4} \) | d) \( \frac{1}{3} \)

5.

(b) is both a chromatic and diatonic transposition.

6. A B A C

Answers will vary, but there are many options!

Examples: M1 beat 1 is rhythmically and melodically translated for the rest of measure.

M1 is translated to M2.

M2s 1-2 are diatonically transposed up 1 step in M2s 5-6.

M3s 3-4 are chromatically transposed up 1 whole step in M3s 7-8.
7. a) G  b) D\textsuperscript{7}  c) A\textsuperscript{aug}  d) Gm\textsuperscript{7}  
  e) ii\textsuperscript{7}  f) IV\textsuperscript{2}  g) vii\textsuperscript{0}\textsuperscript{7}  h) I 

8. \[\text{Diagram: Staff with notes} \]

9. a) \[\text{Music notation with accidentals} \]  b) \[\text{Music notation with accidentals} \]  
  c) \[\text{Music notation with accidentals} \]  d) \[\text{Music notation with accidentals} \] 

10. a) minor  b) dominant  
  (c) full diminished  d) minor  e) major 

a) dominant  
  b) minor  
  c) dominant  
  d) dominant  
  e) dominant