Math 109 HW5 Answer Key Fall 2022

1 a) $\mathbb{R}, \cdot$

✓ Associativity: $x, y, z \in \mathbb{R}$

$(x \cdot y) \cdot z = x \cdot (y \cdot z)$

✓ Identity: $1 \cdot x = x = x \cdot 1$ for any $x \in \mathbb{R}$

$e = 1$

✗ Inverse: 0 does not have a multiplicative inverse

Monoid, not a group

b) $\mathbb{Z}, +$

✓ Associativity: $x, y, z \in \mathbb{R}$

$(x+y)+z = x+(y+z)$

✓ Identity: $0 + x = x = x + 0$ for any $x \in \mathbb{Z}$

$e = 0$

✓ Inverse: For any $x \in \mathbb{Z}$, $-x \in \mathbb{Z}$

$x + (-x) = 0 = e$

Monoid & Group

c) $\{1, -1\}, \cdot$

✓ Associativity: $(1 \cdot 1) \cdot 1 = 1 = 1 \cdot (1 \cdot 1)$

$(-1 \cdot 1) \cdot 1 = -1 = -1 \cdot (1 \cdot 1)$

$\vdots$ ← Order of multiplying 1 and -1 doesn't matter

$(-1 \cdot -1) \cdot -1 = -1 = -1 \cdot (-1 \cdot -1)$

✓ Identity: $1 \cdot 1 = 1$, $1 \cdot -1 = -1$

$e = 1$

✓ Inverse: Inverse = $1/x$, so inverse is itself

$1 \cdot 1 = e$, $-1 \cdot -1 = e$

Monoid & Group

d) $\{-1, 0, 1\}, +$

$1 + 1 = 2 \notin \{-1, 0, 1\}$

Neither

2 a) $[7] + [11] + [9]$ in $\mathbb{Z}_{12}$

$= \boxed{[3]}$

b) $[17] - [25] + [37]$ in $\mathbb{Z}_5$

$= \boxed{[4]}$

c) $[13] - [47] + [6]$ in $\mathbb{Z}_2$

$= \boxed{[1]}$

3 a) $15 + 19 = 34 \pmod{12} \Rightarrow$ **10 semitones**

b) $(2 \cdot 10) + 4 = 24 \pmod{12} \Rightarrow$ **0 semitones**

c) $6 \cdot 5 = 30 \pmod{12} \Rightarrow$ **6 semitones**

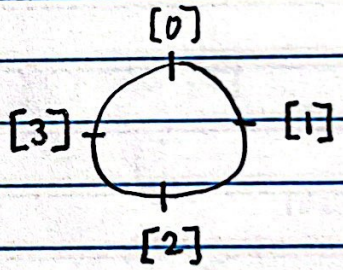
d) $(5 \cdot 4) - (3 \cdot 6) = 2 \pmod{12} \Rightarrow$ **2 semitones**

4 a) $n = 4$

Generators (relatively prime): [1], [3]

$\phi(4) = 2$

Pair: [1] and [3]



b) $n = 5$

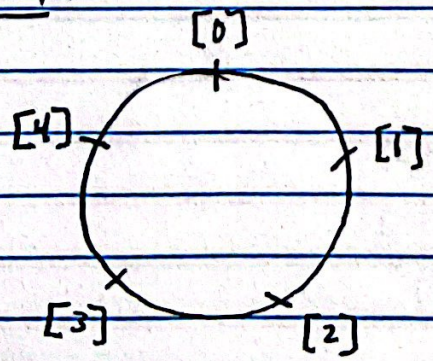
Generators: [1], [2], [3], [4]

$\phi(5) = 4$

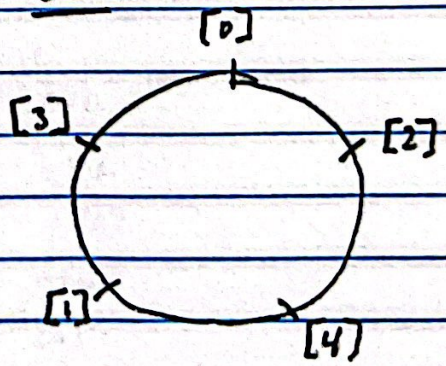
Pairs: [1] and [4]

[2] and [3]

1+4:



2+3:



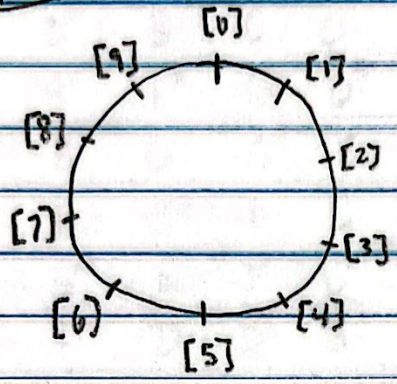
c) $n = 10$

Generators: [1], [3], [7], [9]

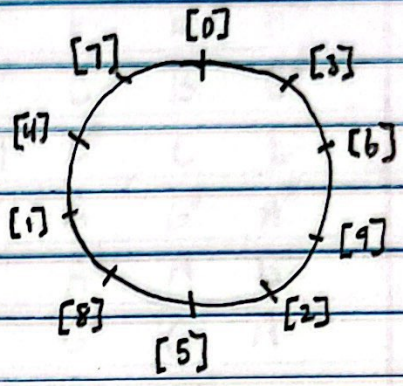
$\phi(10) = 4$

Pairs: [1] and [9], [3] and [7]

1 + 9:



3 + 7:



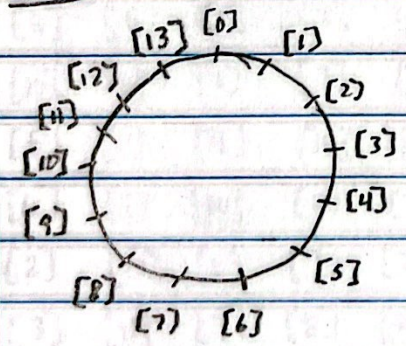
d) $n = 14$

Generators: [1], [3], [5], [9], [11], [13]

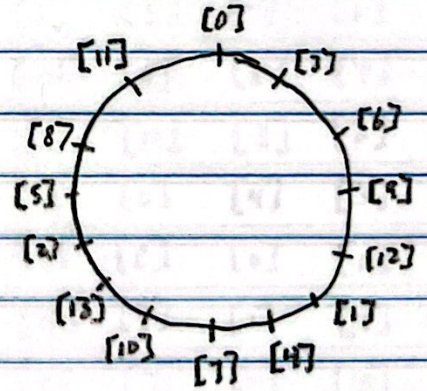
$\phi(14) = 6$

Pairs: [1] and [13], [3] and [11], [5] and [9]

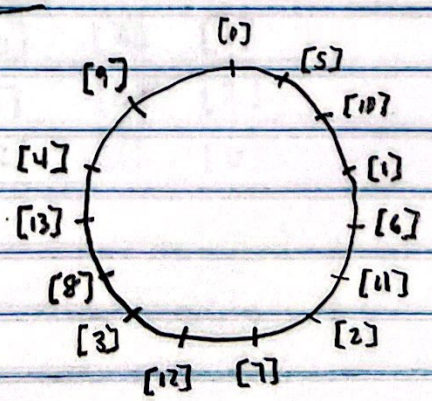
1 + 13:



3 + 11:



5 + 9:



5

A	F	C#	D	A#	C	B	E	D#	G	F#	G#
C#	A	F	F#	D	E	D#	G#	G	B	A#	C
F	C#	A	A#	F#	G#	G	C	B	D#	D	E
E	C	G#	A	F	G	F#	B	A#	D	C#	D#
G#	E	C	C#	A	B	A#	D#	D	F#	F	G
F#	D	A#	B	G	A	G#	C#	C	E	D#	F
G	D#	B	C	G#	A#	A	D	C#	F	E	F#
D	A#	F#	G	D#	F	E	A	G#	C	B	C#
D#	B	G	G#	E	F#	F	A#	A	C#	C	D
B	G	D#	E	C	D	C#	F#	F	A	G#	A#
C	G#	E	F	C#	D#	D	G	F#	A#	A	B
A#	F#	D	D#	B	C#	C	F	E	G#	G	A

6

[0]	[8]	[4]	[5]	[1]	[3]	[2]	[7]	[6]	[10]	[9]	[11]
[4]	[0]	[8]	[9]	[5]	[7]	[6]	[11]	[10]	[2]	[1]	[3]
[8]	[4]	[6]	[1]	[9]	[11]	[10]	[3]	[2]	[6]	[5]	[7]
[7]	[3]	[11]	[0]	[8]	[10]	[9]	[2]	[1]	[5]	[4]	[6]
[11]	[7]	[3]	[4]	[0]	[2]	[1]	[6]	[5]	[9]	[8]	[10]
[9]	[5]	[1]	[2]	[10]	[0]	[11]	[4]	[3]	[7]	[6]	[8]
[10]	[6]	[2]	[3]	[11]	[1]	[0]	[5]	[4]	[8]	[7]	[9]
[5]	[1]	[9]	[10]	[6]	[8]	[7]	[0]	[11]	[3]	[2]	[4]
[6]	[2]	[10]	[11]	[7]	[9]	[8]	[1]	[0]	[4]	[3]	[5]
[2]	[10]	[6]	[7]	[3]	[5]	[4]	[9]	[8]	[0]	[11]	[1]
[3]	[11]	[7]	[8]	[4]	[6]	[5]	[10]	[9]	[1]	[0]	[2]
[1]	[9]	[5]	[6]	[2]	[4]	[3]	[8]	[7]	[11]	[10]	[0]

7 a) $n = 3$

[0]	[2]	[1]
[1]	[0]	[2]
[2]	[1]	[0]

b) $n=4$

[0]	[2]	[3]	[1]
[2]	[0]	[1]	[3]
[1]	[3]	[0]	[2]
[3]	[1]	[2]	[0]

c) $n=6$

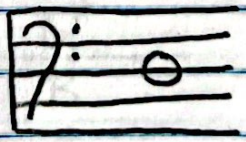
[0]	[2]	[3]	[1]	[4]	[5]
[4]	[0]	[1]	[5]	[2]	[3]
[3]	[5]	[0]	[4]	[1]	[2]
[5]	[1]	[2]	[0]	[3]	[4]
[2]	[4]	[5]	[3]	[0]	[1]
[1]	[3]	[4]	[2]	[5]	[0]

d) $n=7$

[0]	[3]	[1]	[5]	[2]	[6]	[4]
[4]	[6]	[5]	[2]	[6]	[3]	[1]
[6]	[2]	[0]	[4]	[1]	[5]	[3]
[2]	[5]	[3]	[0]	[4]	[1]	[6]
[5]	[1]	[6]	[3]	[0]	[4]	[2]
[1]	[4]	[2]	[6]	[3]	[0]	[5]
[3]	[6]	[4]	[1]	[5]	[2]	[0]

8 $n=2$:

Exact Octave

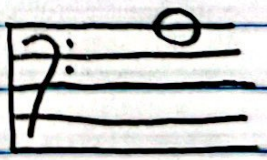


*See Chapter 9

$n=3$:

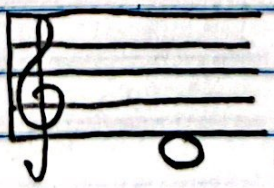
$1200 \log_2 3 \approx 1901.96 \approx$ 2 cents flat

keyboard approx. of 3



$n=4$:

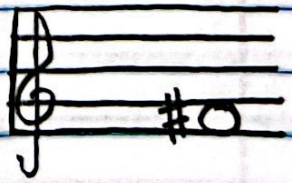
2 Octaves, exact



$n=5$:

$1200 \log_2 5 \approx 2786.31 \approx$ 14 cents sharp

keyboard approx. of 5

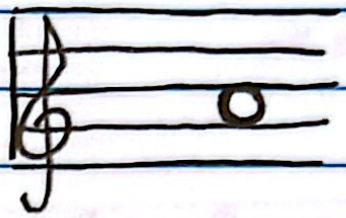


n = 6:

$1200 \log_2 6 = 3101.96 \approx$

keyboard approx. of 6

2 cents flat

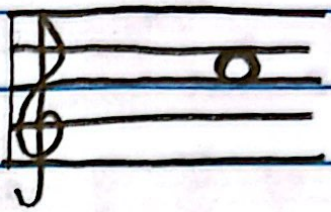


n = 7:

$1200 \log_2 7 \approx 3368.83 \approx$

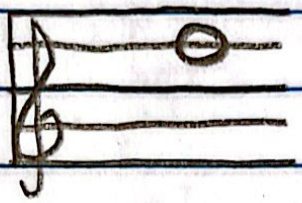
keyboard approx. of 7

31 cents sharp

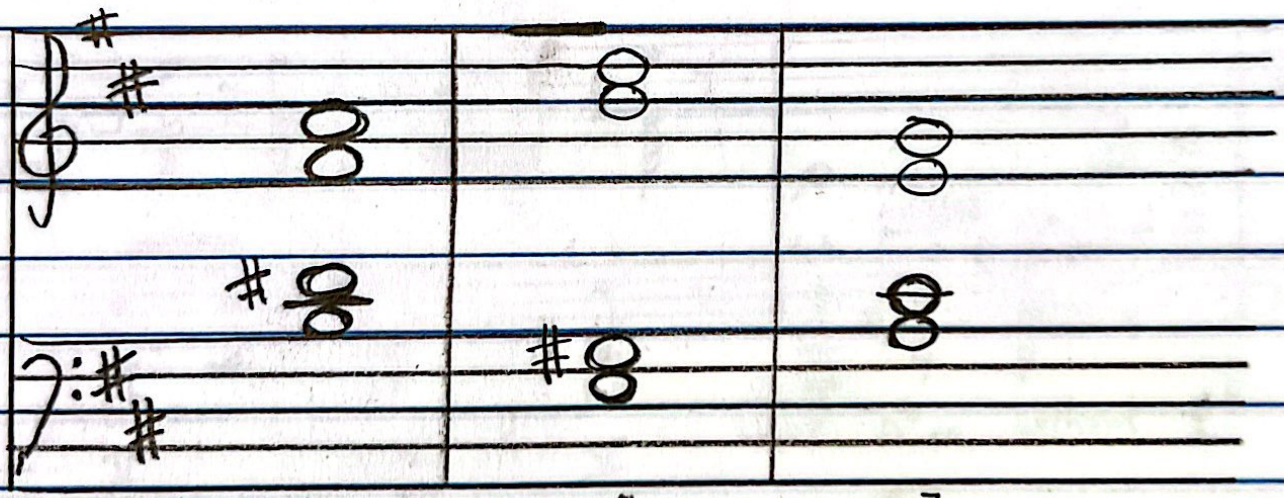


n = 8

3 Octaves, exact



9



D Major

B^7	E^7	A^7
VI^7	II^7	V^7
A	E	A
$F^\#$	$G^\#$	$C^\#$
$D^\#$	B	E
B	D	G

Math 109 HW5 Fall 2022

#10

MAPLE LEAF RAG.

A Flat Major

BY SCOTT JOPLIN.

Tempo di marcia.