

Math 109 HW5 Answer Key Fall 2022

1 a) \mathbb{R}, \cdot

✓ Associativity: $x, y, z \in \mathbb{R}$

$$(x \cdot y) \cdot z = x \cdot (y \cdot z)$$

✓ Identity: $1 \cdot x = x = x \cdot 1$ for any $x \in \mathbb{R}$

$$e = 1$$

✗ Inverse: 0 does not have a multiplicative inverse

Monoid, not a group

b) $\mathbb{Z}, +$

✓ Associativity: $x, y, z \in \mathbb{Z}$

$$(x+y)+z = x+(y+z)$$

✓ Identity: $0+x = x = x+0$ for any $x \in \mathbb{Z}$

$$e = 0$$

✓ Inverse: For any $x \in \mathbb{Z}$, $-x \in \mathbb{Z}$

$$x + (-x) = 0 = e$$

Monoid + Group

c) $\{1, -1\}, \cdot$

✓ Associativity: $(1 \cdot 1) \cdot 1 = 1 = 1 \cdot (1 \cdot 1)$

$$(-1 \cdot 1) \cdot 1 = -1 = -1 \cdot (1 \cdot 1)$$

∴ ← Order of multiplying 1 and -1 doesn't matter

$$(-1 \cdot -1) \cdot -1 = -1 = -1 \cdot (-1 \cdot -1)$$

✓ Identity: $1 \cdot 1 = 1$, $1 \cdot -1 = -1$

$$e = 1$$

✓ Inverse: Inverse = $1/x$, so inverse is itself

$$1 \cdot 1 = e, \quad -1 \cdot -1 = e$$

Monoid + Group

d) $\{-1, 0, 1\}, +$

$$1+1 = 2 \notin \{-1, 0, 1\}$$

Neither

2 a) $[7] + [11] + [9]$ in \mathbb{Z}_{12}
 $= [3]$

b) $[17] - [25] + [37]$ in \mathbb{Z}_5
 $= [4]$

c) $[13] - [47] + [6]$ in \mathbb{Z}_2
 $= [1]$

3 a) $15 + 19 = 34 \pmod{12} \Rightarrow 10$ semitones

b) $(2 \cdot 10) + 4 = 24 \pmod{12} \Rightarrow 0$ semitones

c) $6 \cdot 5 = 30 \pmod{12} \Rightarrow 6$ semitones

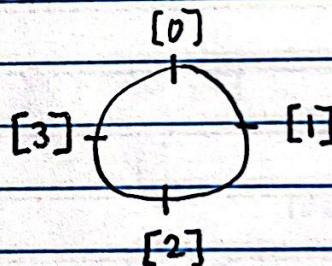
d) $(5 \cdot 4) - (3 \cdot 6) = 2 \pmod{12} \Rightarrow 2$ semitones

4 a) $n = 4$

Generators (relatively prime): $[1], [3]$

$\phi(4) = 2$

Pair: $[1]$ and $[3]$



b) $n = 5$

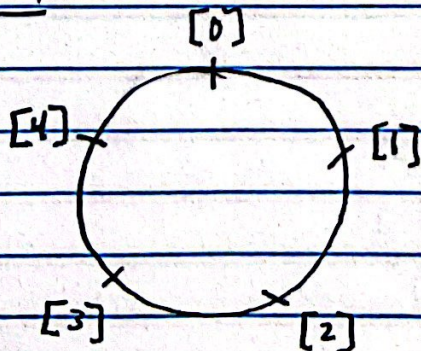
Generators: $[1], [2], [3], [4]$

$\phi(5) = 4$

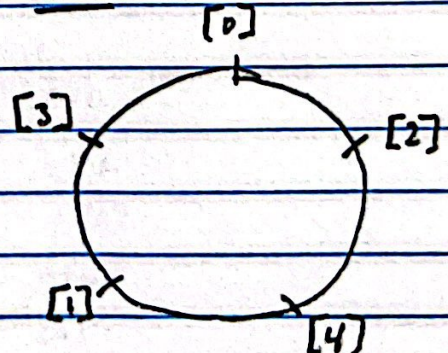
Pairs: $[1]$ and $[4]$

$[2]$ and $[3]$

1+4:



2+3:



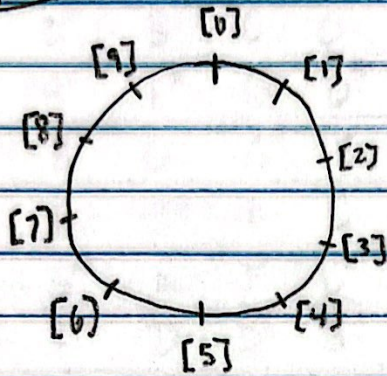
c) $n = 10$

Generators: $[1], [3], [7], [9]$

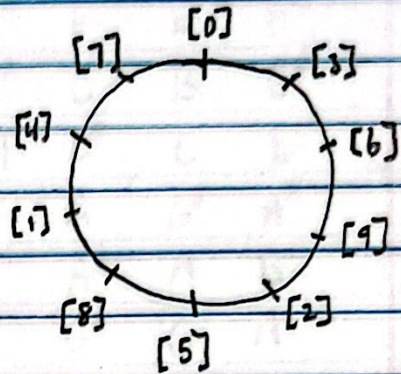
$$\phi(10) = 4$$

Pairs: $[1]$ and $[9]$, $[3]$ and $[7]$

$1+9$:



$3+7$:



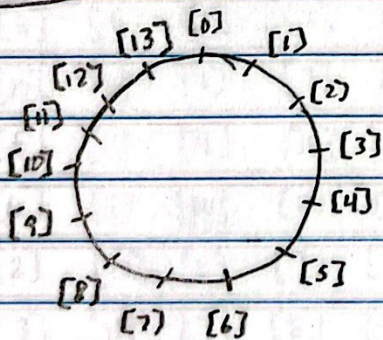
d) $n = 14$

Generators: $[1], [3], [5], [9], [11], [13]$

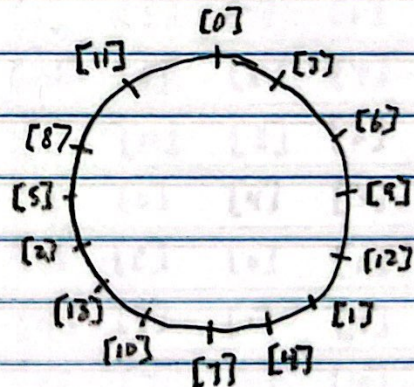
$$\phi(14) = 6$$

Pairs: $[1]$ and $[13]$, $[3]$ and $[11]$, $[5]$ and $[9]$

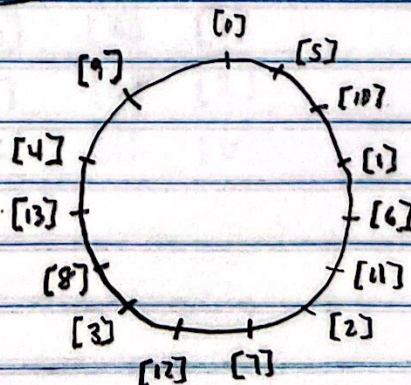
$1+13$:



$3+11$:



$5+9$:



5	A	F	C [#]	D	A [#]	C	B	E	D [#]	G	F [#]	G [#]
	C [#]	A	F	F [#]	D	E	D [#]	G [#]	G	B	A [#]	C
	F	C [#]	A	A [#]	F [#]	G [#]	G	C	B	D [#]	D	E
	E	C	G [#]	A	F	G	F [#]	B	A [#]	D	C [#]	D [#]
	G [#]	F	C	C [#]	A	B	A [#]	D [#]	D	F [#]	F	G
	F [#]	D	A [#]	B	G	A	G [#]	C [#]	C	E	D [#]	F
	G	D [#]	B	C	G [#]	A [#]	A	D	C [#]	F	E	F [#]
	D	A [#]	F [#]	G	D [#]	F	E	A	G [#]	C	B	C [#]
	D [#]	B	G	G [#]	E	F [#]	F	A [#]	A	C [#]	C	D
	B	G	D [#]	E	C	D	C [#]	F [#]	F	A	G [#]	A [#]
	C	G [#]	E	F	C [#]	D [#]	D	G	F [#]	A [#]	A	B
	A [#]	F [#]	D	D [#]	B	C [#]	C	F	E	G [#]	G	A

6	[0]	[8]	[4]	[5]	[1]	[3]	[2]	[7]	[6]	[10]	[9]	[11]
	[4]	[0]	[8]	[9]	[5]	[7]	[6]	[11]	[10]	[2]	[1]	[3]
	[8]	[4]	[6]	[1]	[9]	[11]	[10]	[3]	[2]	[6]	[5]	[7]
	[7]	[3]	[11]	[0]	[8]	[10]	[9]	[2]	[1]	[5]	[4]	[6]
	[11]	[7]	[3]	[4]	[0]	[2]	[1]	[6]	[5]	[9]	[8]	[10]
	[9]	[5]	[1]	[2]	[10]	[0]	[11]	[4]	[3]	[7]	[6]	[8]
	[10]	[6]	[2]	[3]	[11]	[1]	[0]	[5]	[4]	[8]	[7]	[9]
	[5]	[1]	[9]	[10]	[6]	[8]	[7]	[0]	[11]	[3]	[2]	[4]
	[6]	[2]	[10]	[11]	[7]	[9]	[8]	[1]	[0]	[4]	[3]	[5]
	[2]	[10]	[6]	[7]	[3]	[5]	[4]	[9]	[8]	[0]	[11]	[1]
	[3]	[11]	[7]	[8]	[4]	[6]	[5]	[10]	[9]	[1]	[0]	[2]
	[1]	[9]	[5]	[6]	[2]	[4]	[3]	[8]	[7]	[11]	[10]	[0]

7	a)	n = 3	[0]	[2]	[1]
			[1]	[0]	[2]
			[2]	[1]	[0]

b) $n=4$

[0]	[2]	[3]	[1]
[2]	[0]	[1]	[3]
[1]	[3]	[0]	[2]
[3]	[1]	[2]	[0]

c) $n=6$

[0]	[2]	[3]	[1]	[4]	[5]
[4]	[0]	[1]	[5]	[2]	[3]
[3]	[5]	[0]	[4]	[1]	[2]
[5]	[1]	[2]	[0]	[3]	[4]
[2]	[4]	[5]	[3]	[0]	[1]
[1]	[3]	[4]	[2]	[5]	[0]

d) $n=7$

[0]	[3]	[1]	[5]	[2]	[6]	[4]
[4]	[0]	[5]	[2]	[6]	[3]	[1]
[6]	[2]	[0]	[4]	[1]	[5]	[3]
[2]	[5]	[3]	[0]	[4]	[1]	[6]
[5]	[1]	[6]	[3]	[0]	[4]	[2]
[1]	[4]	[2]	[6]	[3]	[0]	[5]
[3]	[6]	[4]	[1]	[5]	[2]	[0]

8 $n=2$:
Exact Octave

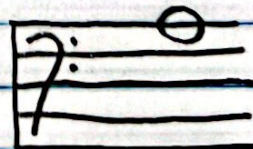


* See Chapter 9

$n=3$:

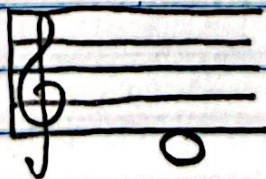
$1200 \log_2 3 \approx 1901.96 \approx$ 2 cents flat

Keyboard approx. of 3



$n=4$:

2 Octaves, exact

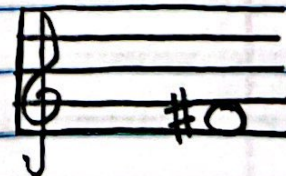


$n=5$:

$1200 \log_2 5 \approx 2786.31 \approx$

Keyboard approx. of 5

14 cents sharp

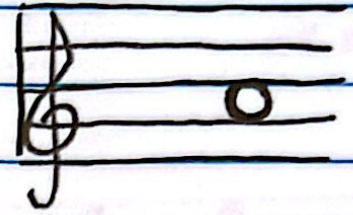


n=6:

$1200 \log_2 6 = 3101.96 \approx$

keyboard approx. of 6

2 cents flat

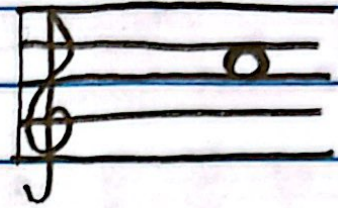


n=7:

$1200 \log_2 7 \approx 3368.83 \approx$

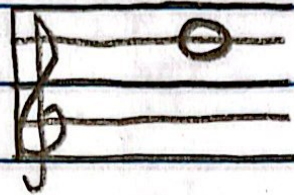
keyboard approx. of 7

31 cents sharp

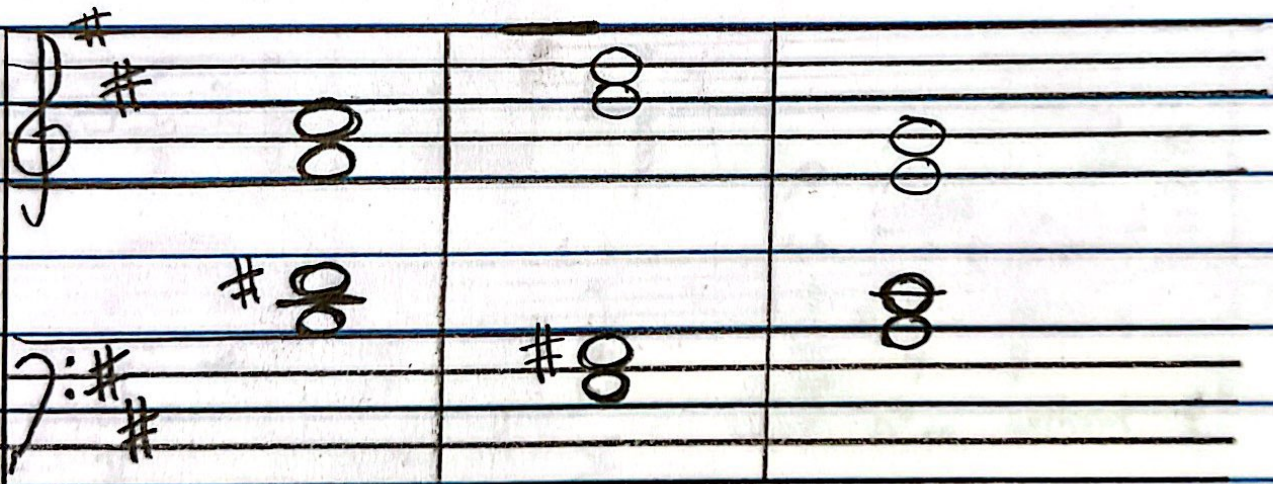


n=8

3 Octaves, exact



9



D Major

B⁷
VI⁷

E⁷
II⁷

A⁷
V⁷

A
F#
D#
B

E
G#
B
D

A
C#
E
G

MAPLE LEAF RAG.

A Flat Major

BY SCOTT JOPLIN.

Tempo di marcia.

The musical score for "Maple Leaf Rag" is presented in four systems. Each system consists of a piano (right hand) staff and a bass (left hand) staff. The key signature is A-flat major (three flats: B-flat, E-flat, A-flat), and the time signature is 2/4. The score includes various musical notations such as slurs, accents, and dynamic markings like *mf*. Handwritten annotations in black ink provide chord symbols for both hands throughout the piece. The first system has chords: I (A-flat), V⁷ (E-flat⁷), I (A-flat), and V⁷ (E-flat⁷). The second system has chords: bVI (F-flat), V (E-flat), bVI (F-flat), V (E-flat), and A^bm without E^b. The third system has chords: #IV⁰⁷ (D dim⁷), I (A-flat), bVI (F-flat), I (A-flat), I (A-flat), V⁷ (E-flat⁷), and I (A-flat). The fourth system has chords: #IV⁰⁷ (D dim⁷), I (A-flat), bVI (F-flat), I (A-flat), I (A-flat), V⁷ (E-flat⁷), and I (A-flat). The score concludes with a first ending (1.) and a second ending (2.).

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