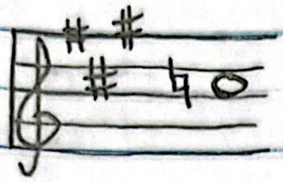
Math 109 HW6 Answer Key Fall 2022

1)

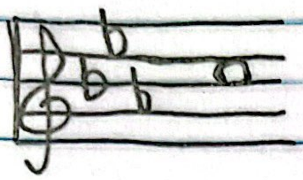
a) 

N: F_2

$k = 6$

$12\log_2(6) \approx 31$ semitones

$\boxed{M: C_5}$

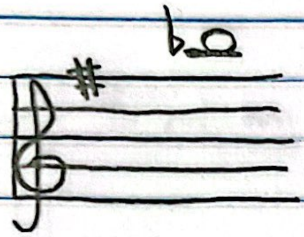
b) 

N: A^b_2

$k = 5$

$12\log_2(5) \approx 28$ semitones

$\boxed{M: C_5}$

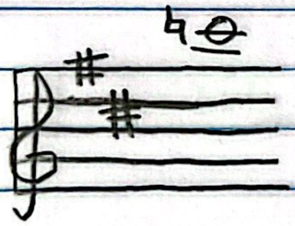
c) 

N: C_3

$k = 7$

$12\log_2(7) \approx 34$ semitones

$\boxed{M: B^b_5}$

d) 

N: E_2

$k = 13$

$12\log_2(13) \approx 44$ semitones

$\boxed{M: C_6}$

2) Major Chords: 2-3-4-5

3-4-5-6

7th Chords: 4-5-6-7

5-6-7-8

Non-Adjacent

Minor: 6-7-9

Diminished: 7-10-12

→ More non-adjacent chords are present. Identify at least two for full credit.

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3) Let $y = f(t)$ have period P

So, $f(t+P) = f(t)$

Let $g(t) = f(t/c)$

Consider $g(t+cP) = f\left(\frac{t+cP}{c}\right)$

$$= f\left(\frac{t}{c} + \frac{cP}{c}\right)$$

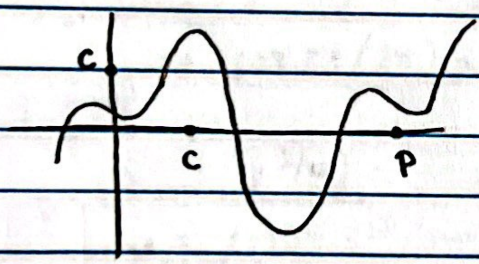
$$= f\left(\frac{t}{c} + P\right)$$

$$= f(t/c)$$

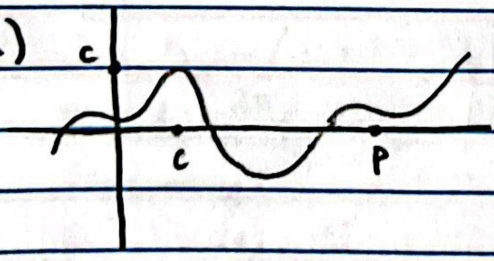
$$= g(t).$$

Thus, $f(t/c)$ has period cP. $\blacksquare$

4)



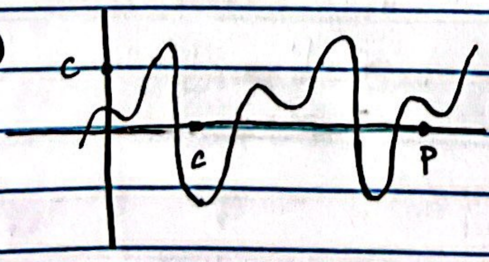
a)



$y = \frac{1}{2} f(t)$

Volume will be $\frac{1}{2}$ of the original

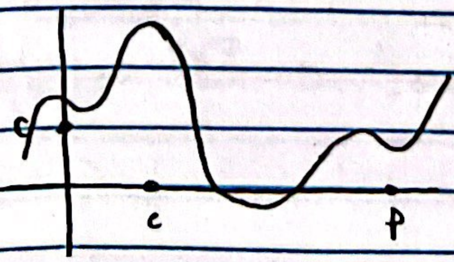
b)



$y = f(2t)$

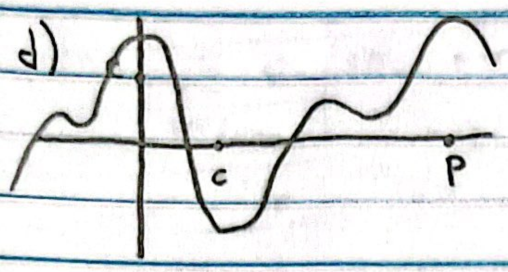
The frequency doubled. Pitch is an octave higher than the original.

c)



$y = f(t) + c$

No effect on pitch or volume

d) 

$y = f(t+c)$

Phase shift alters timing, but pitch and volume stay the same.

5) $A_4 = 440 \text{ Hz}$

$$y = \sin(2\pi f t) \Rightarrow \alpha = 2\pi f$$

a) C_4

$$440\left(2^{-9/12}\right) = 261.62 \text{ Hz}$$

$$\alpha = 261.62 \times 2\pi = \boxed{1643.84}$$

b) A^b_2

$$110\left(2^{-1/12}\right) = 103.83 \text{ Hz}$$

$$\alpha = 103.83(2\pi) = \boxed{652.36}$$

c) $D^{\#}_6$

$$880\left(2^{6/12}\right) = 1244.51 \text{ Hz}$$

$$\alpha = (1244.51)(2\pi) = \boxed{7819.47}$$

6) a) $f(t) = 5\sin(30\pi t + \pi/4)$

Period: $2\pi / 30\pi = 1/15$

Frequency: $1/p = 15$

Amplitude: 5

Phase Shift: $\pi/4$

$$A = d\cos\beta = 5\cos\left(\frac{\pi}{4}\right) = \frac{5\sqrt{2}}{2}$$

$$B = d\sin\beta = 5\sin\left(\frac{\pi}{4}\right) = \frac{5\sqrt{2}}{2}$$

$$\alpha = 30\pi$$

$$f(t) = \frac{5\sqrt{2}}{2}\sin(30\pi t) + \frac{5\sqrt{2}}{2}\cos(30\pi t)$$

b) $g(t) = \sqrt{2}\ \sin(800t + \pi)$

Period: $2\pi/800 = \pi/400$

Frequency: $1/P = 400/\pi$

Amplitude: $\sqrt{2}$

Phase Shift: π

$A = \sqrt{2}\cos(\pi) = -\sqrt{2}$

$B = \sqrt{2}\sin(\pi) = 0$

$g(t) = -\sqrt{2}\ \sin(800t)$

7) a) $f(t) = 4\sin(300t) + 5\cos(300t)$

Period: $2\pi/300 = \pi/150$

Frequency: $150/\pi$

Amplitude: $\sqrt{4^2+5^2} = \sqrt{41}$

Phase Shift: $\beta = \cos^{-1}\left(\frac{A}{\sqrt{41}}\right) = \cos^{-1}\left(\frac{4}{\sqrt{41}}\right) = 0.896$

$f(t) = \sqrt{41}\ \sin(300t + 0.896)$

b) $h(t) = -\sin(1500\pi t) + 3\cos(1500\pi t)$

Period: $2\pi/1500\pi = 1/750$

Frequency: 750

Amplitude: $\sqrt{(-1)^2+3^2} = \sqrt{10}$

Phase Shift: $\beta = \cos^{-1}\left(\frac{-1}{\sqrt{10}}\right) = 1.89$

$h(t) = \sqrt{10}\ \sin(1500\pi t + 1.89)$

8) 5^{th} Harmonic $= 2900$ Hz $= 5 \cdot F$

$F = \frac{2900}{5} = \boxed{580\ \text{HZ}}$

When using $\sin^{-1}$,
Phase Shift $= \pi - \sin^{-1}(3/\sqrt{10})$
due to the quadrant

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9) We are looking for harmonics in the range of
2600 - 3400 Hz

A_2 = 110 Hz (Assuming A_4 is 440 Hz)

Harmonics in Range:

2640 Hz, 2750 Hz, 2860 Hz, 2970 Hz, 3080 Hz, 3190 Hz, 3300 Hz

$E_3 = 110 (2^{7/12}) = 164.81$ Hz

Harmonics in Range:

2637.02 Hz, 2801.83 Hz, 2966.65 Hz, 3131.46 Hz, 3296.28 Hz

Pairs:

2640 + 2637.02

2970 + 2966.65

3300 + 3296.28

Yes, this "near alignment" could be perfected by slightly adjusting the interval. By slightly altering the original frequency, the harmonics will be changed enough to match each other and create a reinforced harmonic.

10 $g(t) = 0.2 \sin(880\pi t) + 0.1 \sin(1760\pi t) + 2 \sin(2640\pi t)$

Fundamental Pitch: $880\pi / 2\pi = \boxed{440 \text{ Hz} \leftarrow A_4}$

The second harmonic (880 Hz ← A_5) will likely not be heard due to low amplitude

The third harmonic will likely be heard due to the large amplitude of 2

1320 Hz is approx. by E_6 (with an error of about 2 cents)