Each problem is worth 10 points.

(1) For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm \( (n = qm + r) \):

(a) 5, 78; \[ 78 = 15 \cdot 5 + 3 \] \[ q = 15, \ r = 3 \]

(b) 7, -21k + 13, where \( k \) some integer.
\[ -21k + 13 = (-3k + 1) \cdot 7 + 6 \] \[ q = -3k + 1,\ r = 6 \]

Sketch the graph of the function \( g(x) = -1 + \cos 2x \) by starting with a more basic function and applying one or more geometric transformations (shifts or stretches).
(3) For the set \( \{(a, b) \in \mathbb{Z}^2 \mid b \neq 0\} \) show that the relation \( \sim \) defined by \( (a, b) \sim (a', b') \) iff \( ab' - a'b = 0 \) is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers \( \mathbb{Q} \).

OR

For the set \( \mathbb{Z} \) and a fixed positive integer \( m \), show that the relation \( \equiv \) defined by \( k \equiv \ell \) iff \( m \mid k - \ell \) is an equivalence relation. Explain why there are exactly \( m \) equivalence classes.

(4) Write the indicated note as a whole note on the given staff, choosing an appropriate clef.

(a) \( \begin{array}{c} \text{c}_3 \end{array} \)

(b) \( \begin{array}{c} \text{c}_\sharp \end{array} \)

(d) \( \begin{array}{c} \text{b} \# \# \end{array} \)

Identify these keyboard intervals:

(c) \( A_4 \) to \( c_5 \) 3 semitones minor third

(d) \( B_4 \) to \( A_4 \) 6 semitones tritone
(5) For the following modes and tonic notes, indicate the appropriate key signature on the given staff:

(a) Phrygian with tonic B

(b) Locrian with tonic E

(6) Transpose this melodic excerpt, written in C minor, up to F♯ minor. Preserve the scale-tone spelling of each melody note.

(7) Give the duration in beats of:

(a) a half note in $\frac{6}{8}$ time (compound time signature).

(b) a dotted eighth note in $\frac{2}{4}$ time.

(c) an eighth note 5-tuplet in $\frac{4}{4}$ time.

(8) On the line below notate and name the following tuplets:

(a) that which divides the half note into 3 equal notes

(b) that which divides the quarter note into 5 equal notes

(c) that which divides the whole note into 11 equal notes
(9) Complete these measures with a single durational note:

(a) \[ \frac{4}{2} \text{ beats} \]

(b) \[ \frac{5}{2} \text{ beats} \]

(c) \[ \frac{12}{2} \text{ beats} \] (compound)

(10) For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

\[ \text{Mary had a little lamb, little lamb,} \]
\[ \text{little lamb, Mary had a little lamb, his} \]
\[ \text{fleece was white as snow} \]

\[ \text{A B A C (or A B A C') } \]
# 3 continued

(a) For \( h \in \mathbb{Z} \) \( m \cdot 0 = k - 1 \cdot 0 \), so \( m \mid (k - k) \). So \( k \equiv k \) (reflexive).

(b) If \( h \equiv k \), then \( m \mid (h - k) \) so \( qm = h - k \) for some \( q \in \mathbb{Z} \).

Multiply by \(-1\) to get \( (-q)m = k - l \), showing \( m \mid (l - k) \), hence \( l \equiv k \). (Symmetric)

(c) Assume \( h \equiv l \) and \( l \equiv r \). Then \( m \mid (l - r) \), so there exist \( p, q \in \mathbb{Z} \) with \( pm = h - l \), \( qm = l - r \).

Add to get \( pm + qm = h - l + l - r \), i.e. \( (p + q)m = h - r \).

This shows \( m \mid (h - r) \) so \( h \equiv r \) (transitive).

Therefore \( \equiv \) is an equivalence relation.

For any \( u \in \mathbb{Z} \), write \( n = qm + r \) with \( 0 \leq r < m \) (Div. Alg.). This shows \( m \mid (n - r) \), so \( n \equiv r \), i.e. \( \Sigma r \) = \( \Sigma r \). Hence \( \Sigma 0 \), \( \Sigma 1 \), \( \Sigma m - 1 \) are all the equivalence classes. Moreover if \( 0 \leq r < r' \leq m - 1 \) then \( m \) does not divide \( r' - r \) (\( \Sigma r \) too small) so \( \Sigma r \neq \Sigma r' \). This shows the \( m \) classes \( \Sigma 0 \), \( \Sigma 1 \), \( \Sigma m - 1 \) are distinct.