EXAM I
Math 109 / Music 109A, Spring 2010

Name ___________________________ Id ___________________________

Each problem is worth 10 points.

1. **Aural:** Notate the rhythm (one measure each).

   (a) \[
   \begin{array}{l}
   \text{\textbf{\textless}} \\
   \text{\textbf{\textless}} \\
   \text{\textbf{\textless}} \\
   \text{\textbf{\textless}} \\
   \end{array}
   \]

   (b) \[
   \begin{array}{l}
   \text{\textbf{\textless}} \\
   \text{\textbf{\textless}} \\
   \text{\textbf{\textless}} \\
   \text{\textbf{\textless}} \\
   \end{array}
   \]

   Circle the triad type.

   (c) major
   (d) minor

   diminished

   diminished

2. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

   (a) \( f(x) = 2x^2 - 2 \)

   (b) \( g(x) = 1 + \sin \frac{3}{2}x \)

3. For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm \( (n = qm + r) \):

   (a) \( 7, -16 \)

   \(-16 = -3 \cdot 7 + 5 \quad \therefore q = -3, \quad r = 5\)

   (b) \( 745k + 18, 5 \) (where \( k \) is some integer)

   \[
   745k + 18 = (149k + 3) 5 + 3 \quad \therefore q = 149k + 3, \quad r = 3
   \]
4. Write the indicated note as a whole note, choosing and notating an appropriate clef.

(a) \[\text{A}_5\]  
(b) \[\text{F}^2_2\]  
(c) \[\text{B}^4_4\]

5. For the set \(Z\) and a fixed positive integer \(m\), show that the relation \(\equiv\) defined by \(k \equiv l\) if and only if \(m \mid (k - l)\) is an equivalence relation. Explain why there are exactly \(m\) equivalence classes.

(i) Reflexive: \(k - k = 0 \cdot m\), so \(m \mid (k - k)\) so \(k \equiv k\)

(ii) Symmetric: if \(k \equiv l\), then \(m \mid (k - l)\) i.e. \(k - l = a \cdot m\) then \(l - k = -a \cdot m\), so \(m \mid (l - k)\) hence \(l \equiv k\).

(iii) Transitive: if \(k \equiv l\) and \(l \equiv r\), then \(m \mid (k - l)\) and \(m \mid (l - r)\) so \(k - l = a \cdot m\) and \(l - r = b \cdot m\). Adding these equations gives \(k - r = a \cdot m + b \cdot m\) i.e. \(k - r = (a + b) \cdot m\) so \(m \mid (k - r)\) and \(k \equiv r\).

Note that \(\text{Z}_0, \ldots, \text{Z}_{m - 1}\) are distinct classes since the difference between any two of the integers \(0, \ldots, m - 1\) is too small to be divisible by \(m\). For any \(n \in Z\), write \(n = q \cdot m + r\) using the division algorithm. Since \(n - r = q \cdot m\) we have \(n \equiv r\), so \(\text{Z}_r = \{r\}\), and hence \(\text{Z}_0 \subseteq \text{Z}_0, \ldots, \text{Z}_{m - 1}\), which shows \(\text{Z}_0, \ldots, \text{Z}_{m - 1}\) are all the classes.
6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Phrygian with tonic D

(c) Aeolian with tonic B⁹

7. Identify each chord in this minor mode (Aeolian) passage. Above the staff label each chord by root note class with suffix (e.g., E⁷). Below the staff, label each chord by root scale tone (e.g. bIII⁷). Also, one of the chords could be considered misspelled. Which chord is it?

8. Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic up one scale tone in the second measure

(b) chromatic up a major third (from the original) in the third measure
9. Give the total duration in beats of:

(a) a doubly-dotted quarter note in \(\frac{3}{4}\) time.
\[d_1 = 1, \quad d_2 = 1 \cdot \left(2 - \frac{1}{4}\right) = \frac{7}{4}\]

(b) a half note in \(\frac{6}{8}\) time (compound time signature).
\[d_1 = \frac{2}{3}, \quad d_2 = 2 \cdot \frac{2}{3} = \frac{4}{3}\]

(c) an eighth note quintuplet in \(\frac{4}{4}\) time.
\[\frac{1}{8} \cdot \frac{1}{2^5} = \frac{1}{2^5} \quad \text{so } n + r = 3 \quad \Rightarrow n = 1 \quad \frac{1}{2} = \text{half note} = \frac{2}{\text{beats}}\]

10. For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

Mary had a little lamb, little lamb, little lamb,

Mary had a little lamb, his fleece was white as snow.

\[\text{A B A C (2 bar segments)}\]

Musical translation: m. 2 \(\rightarrow\) m. 3 \(\rightarrow\) m. 4