EXAM I

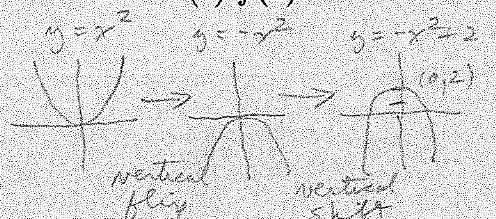
Math 109 / Music 109A, Spring 2018

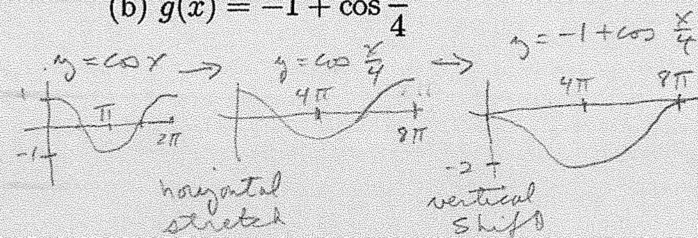
Name Solutions Id ____________

Each problem is worth 10 points.

1. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

   (a) $f(x) = -x^2 + 2$ (b) $g(x) = -1 + \cos\frac{x}{4}$





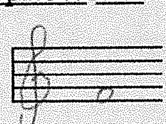
2. For the following pairs of integers m, n, find the numbers q and r whose existence is asserted in the division algorithm ($n = qm + r$):

   (a) $7, -22$; $-22 = -4 \cdot 7 + 6$ $\boxed{q = -4,\ r = 6}$

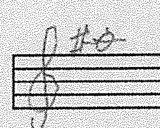
   (b) $3, 102\ell + 4$, where ℓ some integer. $102\ell + 4 = (34\ell + 1) \cdot 3 + 1$

   $\boxed{q = 34\ell + 1,\ r = 1}$

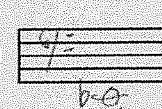
3. Write the indicated note as a whole note, choosing and notating an appropriate clef.

(a) 

F_4

(b) 

$A_5^\sharp$

(c) 

$E_2^\flat$

4. For the set $\{(a,b) \in \mathbb{Z}^2 \,|\, b \neq 0\}$ show that the relation $\sim$ defined by $(a,b) \sim (a',b')$ iff $ab' - a'b = 0$ is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers $\mathbb{Q}$.

<u>OR</u>

For the set $\mathbb{Z}$ and a fixed positive integer m, show that the relation $\equiv$ defined by $k \equiv \ell$ iff $m \,|\, (k - \ell)$ is an equivalence relation. Explain why there are exactly m equivalence classes.

$(a,b) \sim (a',b')$ means $ab' - a'b = 0$, or equivalently $ab' = a'b$ which is $\frac{a}{b} = \frac{a'}{b'}$, since $b, b' \neq 0$

reflexive: $\frac{a}{b} = \frac{a}{b}$, so $(a,b) \sim (a,b)$

symmetric: assume $(a,b) \sim (a',b')$. Then $\frac{a}{b} = \frac{a'}{b'}$, so $\frac{a'}{b'} = \frac{a}{b}$ hence $(a',b') \sim (a,b)$

Transitive: Assume $(a,b) \sim (a',b')$ and $(a',b') \sim (a'',b'')$. Then $\frac{a}{b} = \frac{a'}{b'}$ and $\frac{a'}{b'} = \frac{a''}{b''}$. So $\frac{a}{b} = \frac{a''}{b''}$, hence $(a,b) \sim (a'',b'')$.

The function that sends the class of (a,b) to $\frac{a}{b}$ gives a 1-1 correspondence between classes and elements of $\mathbb{Q}$

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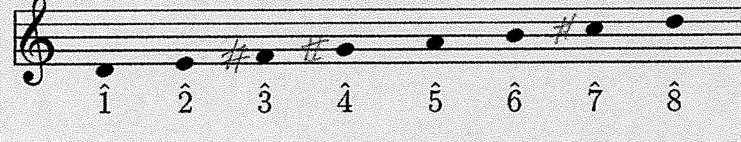
reflexive: $k - k = 0 \cdot m$, so $k \equiv k$.

Symmetric: If $k \equiv l$, then $k - l = am$. Then $l - k = (-a) \cdot m$ so $l \equiv k$.

Transitive: Assume $k \equiv l$ and $l \equiv t$. Then $k - l = am$, $l - t = bm$. Adding these equations gives $k - t = (a+b)m$ so $k \equiv t$.

claim $[0], [1], \ldots, [m-1]$ are all the classes: Given $n \in \mathbb{Z}$, write $n = qm + r$ $0 \leq r < m$. Then $n \equiv r$, so $[n] = [r]$ is one of the classes listed. If $0 \leq r < r' < m$, then $r' - r$ is too small to be divisible by m, so $[r] \neq [r']$. These m classes are distinct.

5. Add the needed sharps or flats <u>to notes</u> so that the following gives the <u>Lydian</u> scale tones $\hat{1}$ to $\hat{8}$, from D to D. (Do <u>not</u> alter D; do <u>not</u> write in a key signature.)



$\hat{1}$ $\hat{2}$ $\hat{3}$ $\hat{4}$ $\hat{5}$ $\hat{6}$ $\hat{7}$ $\hat{8}$

6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

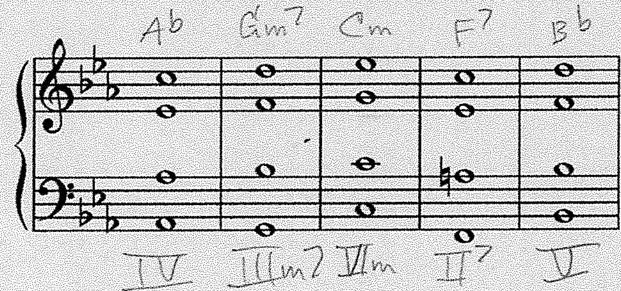
(a) Phrygian with tonic D

same as B♭ major

(c) Aeolian with tonic G♯

same as B major

7. Identify each chord in this major mode (Ionian) passage. Above the staff label each chord by root note class with suffix (e.g., $B^{\flat 7}$). Below the staff, label each chord by root scale tone (e.g. $\flat III^7$).



8. Extend the following melody with two measures having the <u>same rhythm</u>, employing the following transformations. Do not write in a key change.

(a) diatonic up two scale tones in the second measure

(b) chromatic up a major third (from the original) in the third measure

9. Give the (total) duration in beats of:

   (a) a doubly-dotted quarter note in $\frac{3}{2}$ time.

   quarter note has duration $\frac{1}{2}$. $\frac{1}{2}(1+\frac{1}{2}+\frac{1}{4}) = \frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \boxed{\frac{7}{8} \text{ beat}}$

   (b) a sixteenth note in $\frac{9}{8}$ time (compound time signature).

   ♩. = 1 beat so 𝅘𝅥𝅯 = $\boxed{\frac{1}{6} \text{ beat}}$

   (c) a quarter note quintuplet in $\frac{4}{4}$ time.

   $2^2 < 5 < 2^3$ so $r = 2$

   $\frac{1}{2^{n+r}} = \frac{1}{2^2}$ so $n = 0$. Same duration as $\frac{1}{2^0}$-note = whole note so $\boxed{4 \text{ beats}}$

10. For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

A B A C

m2, m3 diatonic & chromatic transposition

m2, m3, m4 and m1, m5, m7 have rhythmic translation