EXAM I
Math 109 / Music 109A, Spring 2020

Name Solutions Id

Each problem is worth 10 points.

1. **Aural:** Circle the triad type.
   
   (a) major
   
   (b) minor

2. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

   (a) \( f(x) = x^2 + 1 \)

   (b) \( g(x) = -1 + \sin 3x \)

3. For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm \((n = qm + r)\):

   (a) \( 11, -23 \) \( \quad -23 = -3 \cdot 11 + 10 \quad q = -3, \quad r = 10 \)

   (b) \( 3, 42d + 5 \) (where \( d \) is some integer) \( 42d + 5 = (14d + 1)3 + 2 \)

4. Write the indicated note as a whole note, choosing and notating an appropriate clef.

   (a) \[ \text{B}_2 \]

   (b) \[ \text{G}^5 \]

   (c) \[ \text{E}^4 \]
5. For the set $\mathbb{Z}$ and a fixed positive integer $m$, show that the relation $\equiv$ defined by $k \equiv \ell$ if and only if $m \mid (k - \ell)$ is an equivalence relation. Explain why there are exactly $m$ equivalence classes.

(a) Given $k \in \mathbb{Z}$, we have $0 \cdot m = 0 = k - k$. Thus $k \equiv k$. This shows $\equiv$ is reflexive.

(b) Suppose $k \equiv \ell$. Then $m \mid (k - \ell)$ so we can write $k - \ell = m \cdot n$ for some $n \in \mathbb{Z}$. Multiplying $k - \ell$ yields $k - k = m (-n)$, which shows $m \mid (k - k)$, hence $k \equiv k$. Thus $\equiv$ is symmetric.

(c) Suppose $k \equiv \ell$ and $\ell \equiv \varphi$. Then $m \mid (k - \ell)$ and $m \mid (\ell - \varphi)$. Write

\[ k - \ell = m \cdot n \quad \text{and} \quad \ell - \varphi = m \cdot q. \]

Adding these equations yields

\[ k - \ell + \ell - \varphi = mn + mq \]

\[ k - \varphi = m(n + q) \]

Hence $k \equiv \varphi$ and $\equiv$ is transitive.

The equivalence classes $[0], [1], \ldots, [m-1]$ are distinct, since if $0 \leq k < \ell \leq m - 1$, $\ell - k$ is too small to be divisible by $m$. Also, for any $k \in \mathbb{Z}$, we have $k = qm + r$ with $0 \leq r < m$. So $k - r = qm$ and $[k - r] = [r]$, with $[r]$ being in the listed above. So there are exactly $m$ classes.
6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Lydian with tonic D

(c) Phrygian with tonic C♯

7. Identify each chord in this minor mode (Aeolian) passage. Above the staff label each chord by root note class with suffix (e.g., E♭7). Below the staff, label each chord by root scale tone (e.g. bIII♯).

8. Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic up two scale tones in the second measure

(b) chromatic up a major third (from the original) in the third measure
9. Give the total duration in beats of:

(a) a doubly-dotted quarter note in $\frac{2}{2}$ time. 
\[ d = \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{4} \right) = \frac{1}{2} \left( \frac{4}{4} + \frac{2}{4} + \frac{1}{4} \right) = \frac{7}{8} \]

(b) a half note in $\frac{9}{8}$ time (compound time signature). 
\[ \frac{9}{8} \text{ note} = \frac{1}{2} \text{ beat} \Leftrightarrow \frac{9}{8} \frac{1}{2} = \frac{9}{16} \text{ beat} \]

(c) an eighth note quintuplet in $\frac{4}{4}$ time. 
\[ \frac{1}{2} \text{ note} + r = \frac{1}{2} \text{ note} = \frac{1}{3} \text{ note} \quad \text{so} \quad \frac{1}{2} \frac{1}{3} = \frac{1}{6} \quad \text{half note} = \frac{1}{2} \text{ beat} \]

10. For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

\[
\begin{align*}
\text{Mary had a little lamb, little lamb, little lamb, little lamb,} \\
\text{Mary had a little lamb, his fleece was white as snow.}
\end{align*}
\]

\[ A \ B \ A \ C \ (m. A \ B \ A \ C) \]

rhythmic translation ms. 2, 3, 4 also ms. 1, 7
metric translation ms. 2, 3