EXAM I

Math 109 / Music 109A, Spring 2020

Name Solutions Id ______________

Each problem is worth 10 points.

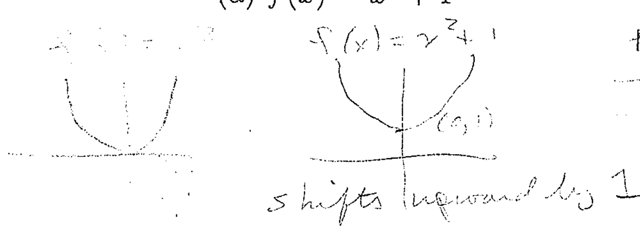
1. Aural: Circle the triad type.

   (a) major
   minor

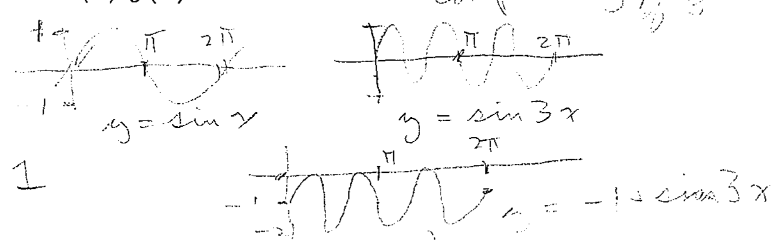
   (b) major
   minor

2. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

   (a) $f(x) = x^2 + 1$



   (b) $g(x) = -1 + \sin 3x$ compressed by factor of 3

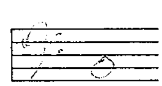


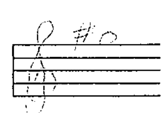
3. For the following pairs of integers m, n, find the numbers q and r whose existence is asserted in the division algorithm ($n = qm + r$):

   (a) $11, -23$ $\quad -23 = -3 \cdot 11 + 10 \quad q = -3, \; r = 10$

   (b) $3, 42d + 5$ (where d is some integer) $\quad 42d + 5 = (14d + 1)3 + 2$
   $q = 14d + 1, \; r = 2$

4. Write the indicated note as a whole note, choosing and notating an appropriate clef.

   (a) 
   B_2

   (b) 
   $G^\sharp_5$

   (c) 
   $E^\flat_4$

5. For the set $\mathbb{Z}$ and a fixed positive integer m, show that the relation $\equiv$ defined by $k \equiv \ell$ if and only if $m \mid (k - \ell)$ is an equivalence relation. Explain why there are exactly m equivalence classes.

(i) Given $k \in \mathbb{Z}$, we have $0 \cdot m = 0 = k - k$. Thus $k \equiv k$. This shows $\equiv$ is reflexive.

(ii) Suppose $k \equiv l$. Then $m \mid (k - l)$ so we can write $k - l = m \cdot n$ for some $n \in \mathbb{Z}$. Multiplying by -1 yields $l - k = m(-n)$ which shows $m \mid (l - k)$, hence $l \equiv k$. Thus $\equiv$ is symmetric.

(iii) Suppose $k \equiv l$ and $l \equiv p$. Then $m \mid (k - l)$ and $m \mid (l - p)$. Write

$k - l = m \cdot n$ and $l - p = m \cdot q$.

Adding these equations yields

$$k - l + l - p = mn + mq$$

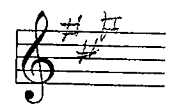
$$k - p = m(n + q)$$ which shows $m \mid (k - p)$

Hence $k \equiv p$ and $\equiv$ is transitive

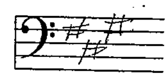
The equivalence classes $[0], [1], \ldots, [m-1]$ are distinct, since if $0 \le k < l \le m-1$, $l - k$ is too small to be divisible by m. Also for any $n \in \mathbb{Z}$, we have $n = qm + r$ with $0 \le r < m$. So $n - r = qm$ and $[n] = [r]$, with $[r]$ being in the list above. So there are exactly m classes.

6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Lydian with tonic D

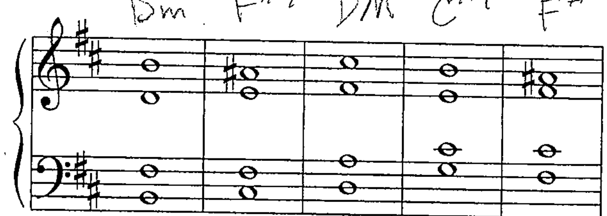


(c) Phrygian with tonic C$^\sharp$



7. Identify each chord in this minor mode (Aeolian) passage. Above the staff label each chord by root note class with suffix (e.g., $E^{\flat 7}$). Below the staff, label each chord by root scale tone (e.g. $\flat III^7$).

Bm $F^{\#7}$ DM^7 $C^{\#\o 7}$ $F^{\#}$

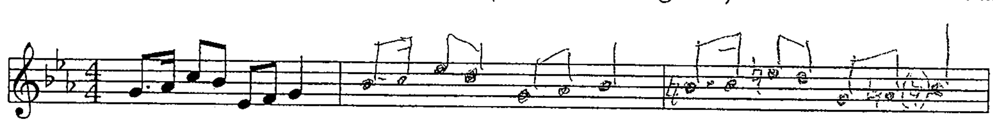


Im V^7 $IIIM^7$ $II^{\o 7}$ V

8. Extend the following melody with two measures having the <u>same rhythm</u>, employing the following transformations. Do not write in a key change.

(a) diatonic up two scale tones in the second measure

(b) chromatic up a major third (from the original) in the third measure



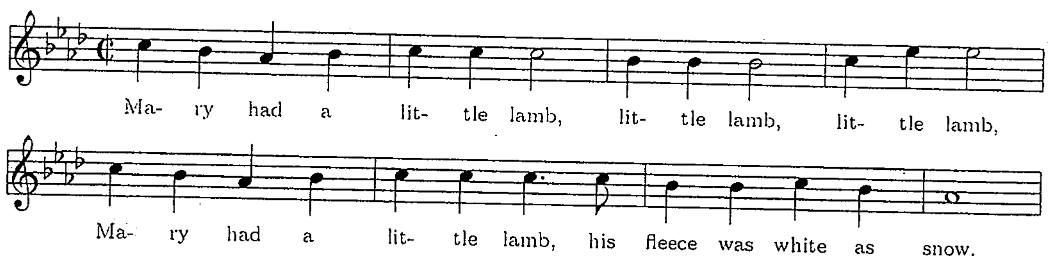
9. Give the total duration in beats of:

   (a) a doubly-dotted quarter note in $\frac{2}{2}$ time. $d = \frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{4}\right) = \frac{1}{2}\left(\frac{4+2+1}{4}\right) = \frac{1}{2}\cdot\frac{7}{4} = \boxed{\frac{7}{8}}$

   (b) a half note in $\frac{9}{8}$ time (compound time signature).
   ♪♪♪ = 1 beat so 𝅗𝅥 = ♪♪♪♪ = $\boxed{\frac{4}{3}\text{ beats}}$

   (c) an eighth note quintuplet in $\frac{4}{4}$ time.
   $\frac{1}{2^{n+r}} = \frac{1}{8} = \frac{1}{2^3}$ $\quad n+r=3 \quad 2^2 < 5 < 2^3$ so $r=2$
   $n+2=3$ so $n=1 \quad \frac{1}{2^1} = \frac{1}{2}$ half note $= \boxed{2\text{ beats}}$

10. For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.



A B A C (or A B A′ C)
rhythmic translation ms. 2, 3, 4 also ms 1, 7
diatonic transposition ms. 2, 3