EXAM II

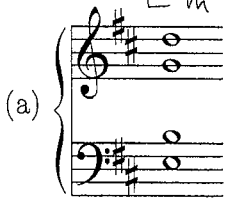
Math 109 / Music 109A, Fall 2022

Name Solutions Id _____

Each problem is worth 10 points. Round off each decimal approximation to two digits to the right of the decimal.

1. Identify these chords by root note with suffix (e.g., Gm^7 or $B^\flat$ aug) and by root scale note with suffix (e.g., III^7 or $\sharp IV\,m$) relative to the indicated mode. to be the lowest note.

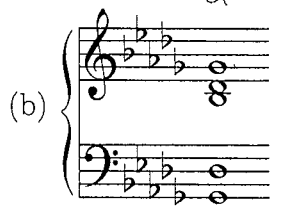
(a) Em7



major

II m7

(b) Gb

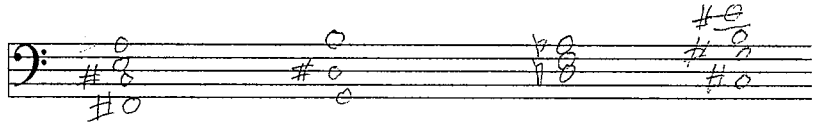


Lydian

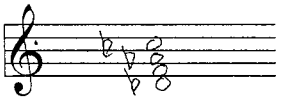
I

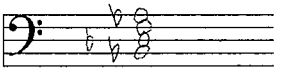
2. Write these chords with correct spelling on the bass clef below.

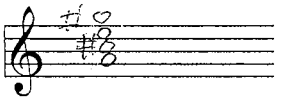
(a) $F^\sharp m^7$ (b) G aug (c) $D^\flat$ (d) $C^{\sharp 7}$



3. Write these chords with correct spelling on the given clef, using the indicated key signature and mode. You may notate the key signature if you wish, but this is not required.

(a) $\flat\text{II}^7$ in the key of C major 

(b) VIm^7 in the key of D minor 

(c) IM^7 in the key of A Myxolydian 

4. Express each of these musical intervals as an element of $\mathbb{R}^+$ three ways: (1) as a power of 2, (2) as a radical or the reciprical of a radical, and (3) by a decimal approximation.

(a) down 46 cents $2^{-46/1200}$ $\frac{1}{\sqrt[1200]{2^{46}}}$ ≈ 0.97

(b) up a minor sixth $2^{8/12} = 2^{2/3}$ $\sqrt[3]{4}$ ≈ 1.59

5. Convert to the specified additive measurement the intervals given by the following ratios.

(a) 19/16, convert to semitones $12 \log_2 \frac{19}{16} \approx 2.98$

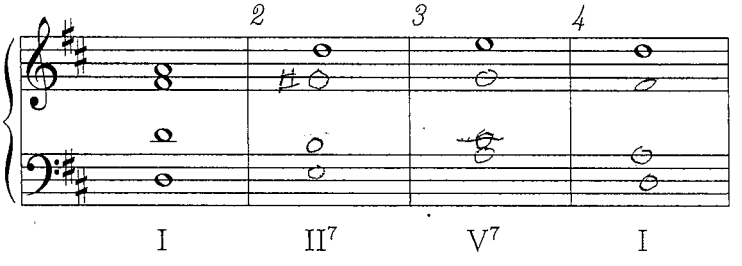
(b) $\pi/6$, convert to cents $1200 \log_2 \frac{\pi}{6} \approx -1120.16$

6. A string on a stringed instrument has length 50 cm. Indicate the positions of the single fret which will allow the string to play the note (a) a keyboard minor third above the original pitch, and (b) a ratio 6/5 with the original pitch.

minor third: $L' = 50 \cdot 2^{-3/12} \approx 42.04$ cm

ratio 6/5: $L' = 50 \cdot \frac{5}{6} \approx 41.67$ cm

7. Complete the following to a four-part harmonization of the given melody, major mode, using only whole notes, so that the melody is the top part, the lowest note is always the root, and the result has two parts on each staff. The chords should be the those indicated under the staff.



8. Evaluate these logarithms <u>without a calculator</u>. Write down each step of the simplification. You may express your answer as a fraction.

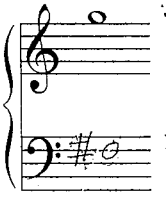
(a) $\log_3\left(\frac{81}{\sqrt{3}}\right) = \log_3 81 - \log_3 \sqrt{3} = \log_3 3^4 - \log_3 3^{1/2}$

$= 4 - \frac{1}{2} = \frac{7}{2}$

(b) $\log_b\left(\frac{b^p}{\sqrt[m]{b^\ell}}\right) = \log_b b^p - \log_b \sqrt[m]{b^\ell} = p - \log_b b^{\ell/m}$

$= p - \frac{\ell}{m} = \frac{mp - \ell}{n}$

9. Write on the staff the keyboard note which best approximates the frequency having the given interval ratio $r = 3/13$ from the given note. Compute the error in cents.



G_5 $\quad 12 \log_2 \frac{3}{13} \approx -25.39$ semitones

$F\#_3$ is ≈ 39 cents above the note with ratio $\frac{3}{13}$ to G_5

10. For the functions $f(x)$ and $g(x)$ defined below, explain how $g(x)$ can be obtained from $f(x)$ by means of one of the standard geometric transformations.

(a) $f(x) = 3^x$, $g(x) = 7^x$ $\quad 7 = 3^{\log_3 7}$ so $g(x) = 3^{x \log_3 7} = f(x \log_3 7)$

Thus $g(x)$ is a horizontal stretch by a factor of $\frac{1}{\log_3 7}$ (compress)

(b) $f(x) = \log_{25} x$, $g(x) = \log_5 x$ $\quad g(x) = \log_5 x = \frac{\log_{25} x}{\log_{25} 5} = \frac{\log_{25} x}{\log_{25} 5^{1/2}}$

$= \frac{\log_{25} x}{\frac{1}{2}} = 2 \log_{25}(x) = 2 f(x)$

Thus $g(x)$ is a vertical stretch by a factor of 2.