

EXAM III

Math 109 / Music 109A, Spring 2018

Name _____ Id _____

Each problem is worth 10 points.

1. Define a (commutative) ring R and show that $0 \cdot x = 0$, for any $x \in R$.
Use this to show that $(-1) \cdot x = -x$, for any $x \in R$.

2. Determine whether these subsets of \mathbb{Z} are ideals. If so, express them in the form $n\mathbb{Z}$, where n is a positive integer. Justify your answers.

(a) the positive integers

(b) $100\mathbb{Z} + 30\mathbb{Z}$

3. Prove that there are infinitely many prime numbers.
4. List the units of the ring \mathbb{Z}_{12} , i.e., the elements of \mathbb{Z}_{12}^* , and explain the implication this has for keyboard intervals.
5. Give the prime factorizations of these integers, writing the primes in ascending order, as in $2^3 \cdot 3^1 \cdot 7^2$.
- (a) 75 (b) 64 (c) 242 (d) 52 (e) 14×10^{23}

6. Prove that if $y = f(t)$ has period P , then $y = cf(t)$ has period P , but $y = f(ct)$ ($c \neq 0$) has period P/c .

7. On the staff system below, write the keyboard's best approximation for harmonics 1 through 13 for the indicated note. For the 7th, 11th, and 13th harmonics, indicate how sharp or flat (to the nearest cent) the keyboard's approximation is.



8. Find the value γ for which the pitch associated to the periodic function $h(t) = d \sin(\gamma t + \beta)$, where t is time in seconds, is E_4^{\flat} .

9. Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = 3 \sin(880\pi t) + 4 \cos(880\pi t)$$

and express it in the form $d \sin(\alpha t + \beta)$, giving a decimal approximation for β .

10. A certain vowel sound has a formant which amplifies frequencies within 350 Hz of 2900 Hz. An alto singer sings the vowel at A_3 . Which harmonics are amplified?