FINAL EXAM

Math 109 / Music 109A, Spring 2018

Name _____ Id _____

Each problem is worth 10 points. Round off each decimal approximation to two digits to the right of the decimal. Your exam should be returned via Crowdmark by Wednesday, May 2. If this timeline is problematic please contact the professor.

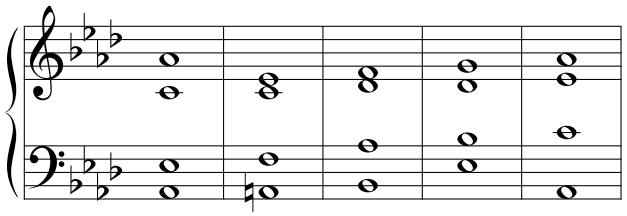
1. Give the (total) duration in beats of:

   (a) a quarter note in $\frac{12}{8}$ time (compound time signature).

   (b) a dotted eighth note in $\frac{3}{2}$ time.

   (c) a quarter note 7-tuplet in $\frac{4}{4}$ time.

2. Identify each chord in this major mode (Ionian) passage. Above the staff label each chord by root note class with suffix (e.g., $E^{\flat 7}$). Below the staff, label each chord by root scale tone (e.g. $\flat III^7$).



3. Convert to semitones the musical intervals given by the following ratios, indicating whether the interval is upward or downward.

   (a) 0.9

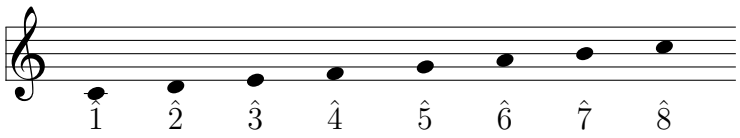
   (b) π

   Express as a ratio the following musical intervals.

   (c) 137 cents

   (d) the just minor third

4. Add the needed sharps or flats <u>to notes</u> so that the following gives the <u>Locrian</u> scale tones $\hat{1}$ to $\hat{8}$, from C to C. Do not alter C, do not write in a key signature.



5. Suppose a 12-tone row chart begins: B, $D^{\flat}$, $A^{\flat}$, E, G, ... Write the upper left 5×5 matrix of the resulting row chart. Then rewrite it replacing each note class with the element of $\mathbb{Z}_{12}$ which measures its modular interval from B.

6. If the keyboard divided the octave into 9 rather than 12 equal intervals, what would be the generating intervals? Quote the relevant facts from modular arithmetic that justify your answer. Draw the circles that exhibit inverse pairs of these generating intervals.

7. (a) Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = \sin(660\pi t) + \sqrt{3}\cos(660\pi t)$$

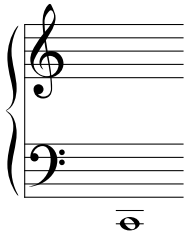
and express it in the form $d\sin(\alpha t+\beta)$, giving a decimal approximation for β. Identify the closest keyboard note to the pitch represented by this function, and the error (if any) in cents.

(b) Find the period, frequency, amplitude, and phase shift for the function

$$h(t) = 3\sqrt{2}\sin\left(1500t + \frac{\pi}{4}\right)$$

and express it in the form $A\sin\alpha t + B\cos\alpha t$. Identify the closest keyboard note to the pitch represented by this function, and the error (if any) in cents. Caution: It says $1500t$, not $1500\pi t$.

8. On the staff system below, write the keyboard's best approximation for each <u>prime</u> harmonic up through 19 for the indicated note, indicating how sharp or flat (in cents) the keyboard's approximation is.



9. The sawtooth wave, defined on $[0, 2\pi)$ by $q(t) = \frac{1}{\pi}t - 1$, has Fourier series

$$q(t) = -\frac{2}{\pi}\left[\sin t + \frac{1}{2}\sin(2t) + \frac{1}{3}\sin(3t) + \frac{1}{4}\sin(4t) + \cdots\right]$$

Give the values of the Fourier coefficients C, A_k, B_k for $k \in \mathbb{Z}^+$, indicate the phase shift of each harmonic, and explain why the value of C is such, based on the graph of $q(t)$.

10. (a) What is the mean-tone fifth? Give its value and explain how it arises. Compare it, in cents, to the just fifth and the fifth of equal temperament.

(b) Compare three just thirds with an octave. Do the same for three Pythagorean thirds. In each case, is it greater than, less than, or equal to an octave, and by how much in cents?

HAVE A GOOD SUMMER!