

Homework 1

Math 109 / Music 109A, Spring 2018

Due Monday, January 29.

1. Identify these notes by letter and subscript (e.g., F_4 or C_2^\sharp):

(a)  (b)  (c)  (d) 

2. Identify these intervals by name (e.g., major third) and by measurement in semitones:

(a)  (b)  (c)  (d) 

3. For the following pairs of integers m, n , find the numbers q and r whose existence is asserted in the Division Algorithm (p. 2 of the text):

(a) $12, 41$;

(b) $9, -28$;

(c) $3, 3^{19} + 7$;

(d) $5, 30k + 7$, where k is some integer.

4. For each of the following sets and relations determine whether or not an equivalence relation has been defined. Explain why or why not. If so, describe the set of equivalence classes.

(a) The set of people having a single residence in the United States; “lives in the same state as”.

(b) $\mathbb{R}; \geq$.

(c) \mathbb{Z} ; for a fixed positive integer n , \equiv defined by $k \equiv \ell$ if and only if $n \mid (k - \ell)$.

(d) The set of keyboard note classes; \sim defined by $N \sim N'$ if and only if the modular interval between N and N' is either the unison interval or a tritone (up or down).

5. For the set $\{(a, b) \in \mathbb{Z}^2 \mid b \neq 0\}$ show that the relation \sim defined by $(a, b) \sim (a', b')$ if and only if $ab' - a'b = 0$ is an equivalence relation and that the set of equivalence classes is in one-to-one correspondence with \mathbb{Q} .
6. Choosing an appropriate clef, write on the staff, and name with subscript, the note which is:

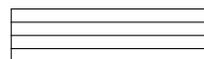
(a) a major third above B_2 .



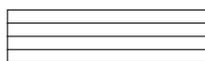
(b) a fourth above G_3^\sharp .



(c) a major ninth below D_6^\sharp .



(d) a tritone below A_4^b .



7. For each of the two sequences of notes below, give the scale tone of each note. Label it above the note by solmization (e.g., *re*) and below it by “hatted” number (e.g., $\hat{2}$). NOTE: The first passage is in the major mode; the second is in the minor mode.



8. For the following modes and tonic notes, indicate the appropriate key signature on staff paper:

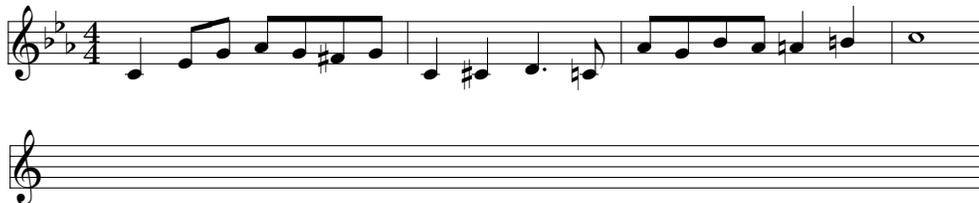
(a) Lydian with tonic C.

(b) Dorian with tonic E^b.

(c) Locrian with tonic G[#].

(d) Phrygian with tonic C.

9. Transpose this melodic excerpt, written in C minor, up to E minor. Write the proper key signature and preserve the scale-tone spelling of each melody note.



10. Sketch the graphs of these functions, and indicate how each is obtained by geometric transformations (shifts and/or stretches) of simpler functions:

(a) $f(x) = 2x - 1$

(b) $f(x) = \frac{1}{2}x^2 + 2$

(c) $f(x) = -1 + 3\sin(2x)$