Homework 3

Math 109 / Music 109A, Spring 2018

Due Monday, February 26.

1. Express each of these intervals as elements of $\mathbb{R}^+$ three ways: (1) as a power of 2, (2) as a radical or the reciprical of a radical, and (3) by a decimal approximation with digits to the right of the decimal.

   (a) up 47 cents

   (b) down 325 cents

   (c) up a minor sixth

   (d) the interval from G_3 to $A_1^\sharp$

2. Assuming A_4 is tuned to 440 Hz, find the frequencies for:

   (a) C_4 (b) $E_2^\flat$ (c) B_5 (d) $A_3^\flat$

   Suppose middle C is tuned as 256 Hz. (Note: This is not standard practice.) Find the frequencies for:

   (a) A_4 (b) $F_5^\flat$ (c) C_1 (d) $G_2^\sharp$

3. For each of these chords, voiced within an octave with the root on the bottom, give the pitch of each note in the chord. Assume A_4 is tuned to 440 Hz.

   (a) major triad with root G_3

   (b) minor triad with root $A_4^\flat$

   (c) minor seventh chord with root D_3

   (d) diminished seventh with root $B_5^\flat$

4. Suppose a string on a banjo has length 50cm. Indicate positions of the 12 frets which will allow the string to play one octave of the ascending chromatic scale.

5. Evaluate without a calculator by writing the argument of log as a power of the base. Write down each step of the simplification, e.g., $\log_3 3\sqrt{3} = \log_3 3^{3/2} = \frac{3}{2}\log_3 3 = \frac{3}{2}$:

   (a) $\log_{10}(0.0001)$ (b) $\log_3 243$ (c) $\log_5 \sqrt[7]{25}$ (d) $\log_b(1/\sqrt[n]{b^\ell})$

   Express as a single logarithm without coefficient, i.e., in the form $\log_b c$ (do not evaluate with a calculator):

   (e) $\log_3 11 + \log_3 17$ (f) $\log_9 5 - 2\log_9 2$

   (g) $\log_2 13 + \log_4 21$ (h) $2\log_c x^2 - \frac{1}{2}\log_{\sqrt{c}} x$

6. Sketch the graphs of:

   (a) $f(x) = 2^x$ (b) $g(x) = \log_2 x$ (c) $r(x) = 5^x$ (d) $s(x) = \log_5 x$

   Determine which pairs of these functions are inverse to each other, and which pairs differ by a horizontal or vertical stretch/compression. In the latter case, identify the stretch factor, justifying your answer.

7. Using laws of exponents, prove this property of logarithms:

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

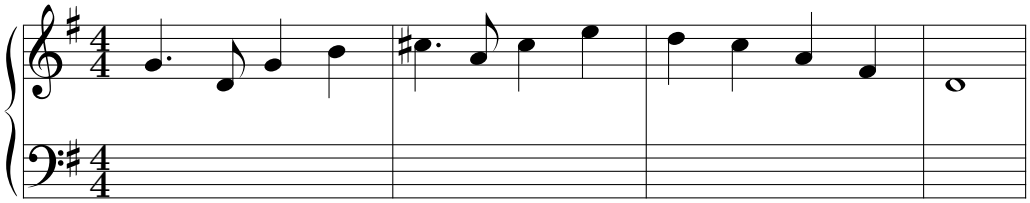
8. Convert to semitones the intervals given by the following ratios: (Round off to 2 digits to the right of the decimal.)

   (a) 7 (b) 0.3 (c) $\frac{5}{2}$ (d) $\sqrt[4]{4}$ (e) e

   Convert to cents the intervals given by the following ratios, rounding off to the nearest whole cent:

   (f) 0.8 (g) 1.2 (h) $\frac{3}{5}$ (i) $\frac{8}{3}$ (j) π

9. Give a plausible harmonization of this melody by providing, in the bass clef, one whole note chord for each measure. Label each chord by root scale tone and chord type (e.g., VIm^7).



10. Analyze the basic harmony in the first 16 measures of *Maple Leaf Rag.* Each measure will have at most two chords. Label the chords by root note class and chord type (e.g., G^7). (Note: In a few places the chords are incomplete.) The music can be downloaded as a pdf file from the website. It is listed under Handouts.