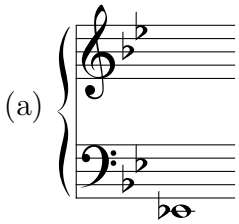
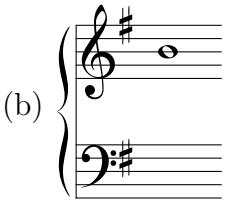
Homework 4

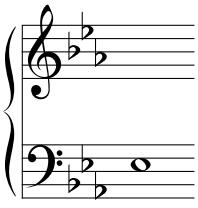
Math 109 / Music 109A, Spring 2018

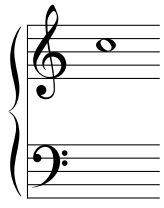
Due Monday, March 19.

1. Write on the staff the note which best approximates the frequency having the given interval ratio r from the given note:

(a)  $r = 5$

(b)  $r = \frac{3}{5}$

(c)  $r = 2.7$

(d)  $r = \frac{2}{\pi}$

2. Express the following interval ratios in terms of n-chromatic units, for the given n. Round off to 2 digits to the right of the decimal.

   (a) ratio $\frac{7}{4}$; $n = 17$

   (b) ratio 3 ; $n = 7$

   (c) ratio 0.54 ; $n = 13$

   (d) ratio e ; $n = 3$ (i.e., major thirds)

3. For the values $n = 11, 19, 23$, find the n-chromatic scale's best approximation of the interval ratio $3/2$, and calculate the error in cents. Which of these values of n gives the best approximation, and is that approximation as good as that of the 12-chromatic scale?

4. Which of the following sets, together with with given operation, form a monoid, and which are also a group? Justify your answers.

   (a) $\mathbb{R}\ , \cdot$ (b) $\mathbb{Z}\ , +$ (c) $\{1, -1\}\ , \cdot$ (d) $\{-1, 0, 1\}\ , +$

5. Show that the functions $f(x) = b^x$ and $g(x) = \log_b(x)$ are group homomorphisms, and that they are inverse to each other, thereby giving isomorphisms between the groups $(\mathbb{R}, +)$ and $(R^+, \cdot)$. Explain how this relates to the measurement of musical intervals.

6. Express the following compositions of modular 12-chromatic intervals as r semitones with $0 \leq r < 12$. Interpret all these compositions as operations in the additive group $\mathbb{Z}_{12}$. (Intervals are upward unless otherwise noted.)

   (a) the composition of 15 and 19 semitones

   (b) two minor sevenths and a major third

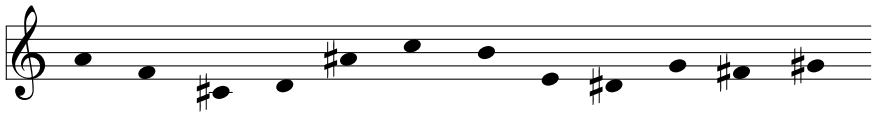
   (c) six fourths

   (d) up five major thirds, down three tritones

7. For each of these choices of n, determine $\phi(n)$ (ϕ is the Euler Phi function) by listing all the generating intervals in the n-chromatic scale. Indicate which pairs of generating intervals are inverse to each other and for each pair draw the circle of intervals which is based on one element of the pair in the clockwise direction, the other element of the pair in the counterclockwise direction.

   (a) n=4 (b) n=5 (c) n=10 (d) n=14

8. Create a twelve-tone row chart having this sequence as its original row:



9. Create n-tone row charts for the following choices of n and the given sequences of original rows in $\mathbb{Z}_n$:

   (a) $n = 3$; $([0], [2], [1])$

   (b) $n = 4$; $([0], [2], [3], [1])$

   (c) $n = 6$; $([0], [2], [3], [1], [4], [5])$

   (d) $n = 7$; $([0], [3], [1], [5], [2], [6], [4])$

10. Analyze the basic harmony in the first five measures of *Moonlight Sonata.* Label the chords by root note class and chord type (e.g., G^7). The music can be downloaded as a pdf file from the website.