Homework 4

Math 109 / Music 109A, Spring 2020

Due Monday, March 16.

NOTE: Express all decimal approximations in this assignment rounded off to three digits to the right of the decimal.

1. Convert to semitones the intervals given by the following ratios:

   (a) 5 (b) 0.4 (c) $\frac{7}{3}$ (d) $\sqrt[4]{6}$ (e) e

   Convert to cents the intervals given by the following ratios:

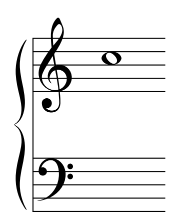
   (f) 0.7 (g) 3.1 (h) $\frac{3}{11}$ (i) $\frac{8}{7}$ (j) $\frac{\pi}{2}$

2. Write on the staff the note which best approximates the frequency having the given interval ratio r from the given note:

(a) 
$r = 5$

(b) 
$r = \frac{3}{5}$

(c) 
$r = 2.7$

(d) 
$r = \frac{2}{\pi}$

3. Express the following interval ratios in terms of n-chromatic units, for the given n.

   (a) ratio $\frac{7}{4}$; $n = 17$

   (b) ratio 3 ; $n = 7$

   (c) ratio 0.54 ; $n = 13$

   (d) ratio e ; $n = 3$ (i.e., major thirds)

4. Evaluate without a calculator by writing the argument of log as a power of the base. Write down each step of the simplification, e.g., $\log_3 3\sqrt{3} = \log_3 3^{3/2} = \frac{3}{2}\log_3 3 = \frac{3}{2}$:

   (a) $\log_{10}(0.001)$ (b) $\log_5 3125$ (c) $\log_3 \sqrt[10]{81}$ (d) $\log_c(1/\sqrt[n]{c^\ell})$

   Express as a single logarithm without coefficient, i.e., in the form $\log_b c$ (do not evaluate with a calculator):

   (e) $\log_4 10 + \log_4 21$ (f) $\log_9 6 - 2\log_9 4$

   (g) $\log_2 13 + \log_4 21$ (h) $2\log_c x^2 - \frac{1}{2}\log_{\sqrt{c}} x$

5. Sketch the graphs of:

   (a) $f(x) = 2^x$ (b) $g(x) = \log_2 x$ (c) $r(x) = 5^x$ (d) $s(x) = \log_5 x$

   Determine which pairs of these functions are inverse to each other, and which pairs differ by a horizontal or vertical stretch/compression. In the latter case, identify the stretch factor, justifying your answer.

6. Using laws of exponents, prove this property of logarithms:

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

7. For the values $n = 11, 19, 23$, find the n-chromatic scale's best approximation of the interval ratio $3/2$, and calculate the error in cents. Which of these values of n gives the best approximation, and is that approximation as good as that of the 12-chromatic scale?

8. Which of the following sets, together with with given operation, form a monoid, and which are also a group? Justify your answers.

   (a) $\mathbb{R}$, $\cdot$ (b) $\mathbb{Z}$, $+$ (c) $\{1, -1\}$, $\cdot$ (d) $\{-1, 0, 1\}$, $+$

9. Express the following compositions of modular 12-chromatic intervals as r semitones with $0 \leq r < 12$. Interpret all these compositions as operations in the additive group $\mathbb{Z}_{12}$. (Intervals are upward unless otherwise noted.)

   (a) the composition of 15 and 19 semitones

   (b) two minor sevenths and a major third

   (c) six fourths

   (d) up five major thirds, down three tritones

10. Analyze the basic harmony in the first five measures of Beethoven's *Moonlight Sonata.* Label the chords by root note class (e.g., V^7) and chord type (e.g., G^7). The music can be downloaded as a pdf file from the website. NOTE: This piece is in the minor mode.