Due Monday, March 16.

NOTE: Express all decimal approximations in this assignment rounded off to three digits to the right of the decimal.

1. Convert to semitones the intervals given by the following ratios:
   
   (a) 5  (b) 0.4  (c) \frac{7}{3}  (d) \sqrt{6}  (e) e

   Convert to cents the intervals given by the following ratios:

   (f) 0.7  (g) 3.1  (h) \frac{3}{11}  (i) \frac{8}{7}  (j) \frac{\pi}{2}

2. Write on the staff the note which best approximates the frequency having the given interval ratio \( r \) from the given note:

   (a) \( r = 5 \)  
   (b) \( r = \frac{3}{5} \)  
   (c) \( r = 2.7 \)  
   (d) \( r = \frac{2}{\pi} \)
3. Express the following interval ratios in terms of \( n \)-chromatic units, for the given \( n \).

(a) ratio \( \frac{7}{4} \); \( n = 17 \)

(b) ratio \( 3 \); \( n = 7 \)

(c) ratio \( 0.54 \); \( n = 13 \)

(d) ratio \( e \); \( n = 3 \) (i.e., major thirds)

4. Evaluate without a calculator by writing the argument of \( \log \) as a power of the base. Write down each step of the simplification, e.g., \( \log_3 3\sqrt{3} = \log_3 3^{3/2} = \frac{3}{2} \log_3 3 = \frac{3}{2} \).

(a) \( \log_{10}(0.001) \)  
(b) \( \log_5 3125 \)  
(c) \( \log_3 \sqrt[3]{81} \)  
(d) \( \log_c(1/\sqrt{c^2}) \)

Express as a single logarithm without coefficient, i.e., in the form \( \log_b c \) (do not evaluate with a calculator):

(e) \( \log_4 10 + \log_4 21 \)  
(f) \( \log_9 6 - 2 \log_9 4 \)

(g) \( \log_2 13 + \log_4 21 \)  
(h) \( 2 \log_c x^2 - \frac{1}{2} \log_{\sqrt{e}} x \)
5. Sketch the graphs of:

(a) \( f(x) = 2^x \)  \qquad (b) \( g(x) = \log_2 x \)  \qquad (c) \( r(x) = 5^x \)  \qquad (d) \( s(x) = \log_5 x \)

Determine which pairs of these functions are inverse to each other, and which pairs differ by a horizontal or vertical stretch/compression. In the latter case, identify the stretch factor, justifying your answer.

6. Using laws of exponents, prove this property of logarithms:

\[
\log_b \frac{x}{y} = \log_b x - \log_b y
\]
7. For the values \( n = 11, 19, 23 \), find the \( n \)-chromatic scale’s best approximation of the interval ratio \( 3/2 \), and calculate the error in cents. Which of these values of \( n \) gives the best approximation, and is that approximation as good as that of the 12-chromatic scale?

8. Which of the following sets, together with with given operation, form a monoid, and which are also a group? Justify your answers.

(a) \( \mathbb{R}, \cdot \)
(b) \( \mathbb{Z}, + \)
(c) \( \{1, -1\}, \cdot \)
(d) \( \{-1, 0, 1\}, + \)
9. Express the following compositions of modular 12-chromatic intervals as \( r \) semitones with \( 0 \leq r < 12 \). Interpret all these compositions as operations in the additive group \( \mathbb{Z}_{12} \). (Intervals are upward unless otherwise noted.)

(a) the composition of 15 and 19 semitones

(b) two minor sevenths and a major third

(c) six fourths

(d) up five major thirds, down three tritones

10. Analyze the basic harmony in the first five measures of Beethoven’s Moonlight Sonata. Label the chords by root note class (e.g., V\(^7\)) and chord type (e.g., G\(^7\)). The music can be downloaded as a pdf file from the website. NOTE: This piece is in the minor mode.