Homework 4

Math 109 / Music 109A, Fall 2022

Due Monday, October 31.

NOTE: Decimal approximations in this assignment should be rounded off to <u>two</u> digits to the right of the decimal.

1. Suppose a string on a banjo has length 50cm. Indicate positions of the 12 frets which will allow the string to play one octave of the ascending chromatic scale.

2. A string on a stringed instrument has length 100 cm. Indicate the positions of the single fret which will allow the string to play the note (a) a keyboard major third above the original pitch, and (b) a ratio 5/4 above the original pitch. (Note the closeness of these two positions.)

3. Evaluate without a calculator by writing the argument of log as a power of the base. Write down each step of the simplification, e.g., $\log_3 3\sqrt{3} = \log_3 3^{3/2} = \frac{3}{2}\log_3 3 = \frac{3}{2}$:

(a) $\log_{10}(0.001)$ (b) $\log_5 3125$ (c) $\log_3 \sqrt[10]{81}$ (d) $\log_c(1/\sqrt[n]{c^\ell})$

Express as a single logarithm without coefficient, i.e., in the form $\log_b c$ (do not evaluate with a calculator):

(e) $\log_4 10 + \log_4 21$ (f) $\log_9 6 - 2\log_9 4$

(g) $\log_2 13 + \log_4 21$ (h) $2\log_c x^2 - \frac{1}{2}\log_{\sqrt{c}} x$

4. Convert to semitones the intervals given by the following ratios: (Round off to 2 digits to the right of the decimal.)

(a) 5   (b) 0.4   (c) $\frac{7}{3}$ (d) $\sqrt[4]{6}$ (e) e

Convert to cents the intervals given by the following ratios, rounding off to the nearest whole cent:

(f) 0.7   (g) 3.1   (h) $\frac{3}{11}$ (i) $\frac{8}{7}$ (j) $\frac{\pi}{2}$
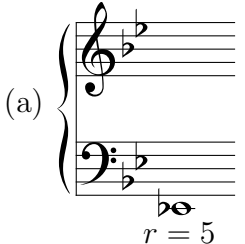
5. Sketch the graphs of:

(a) $f(x) = 2^x$ (b) $g(x) = \log_2 x$ (c) $r(x) = 5^x$ (d) $s(x) = \log_5 x$
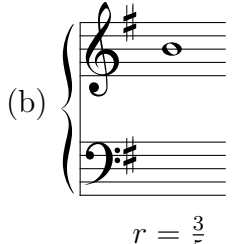
Determine which pairs of these functions are inverse to each other, and which pairs differ by a horizontal or vertical stretch/compression. In the latter case, identify the stretch factor, justifying your answer.
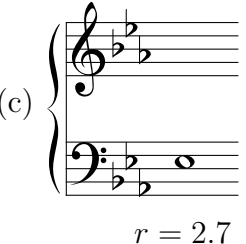
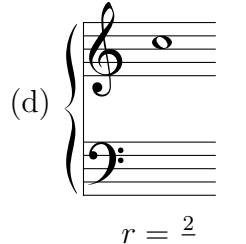6. Using laws of exponents, prove this property of logarithms:

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

7. Write on the staff the note which best approximates the frequency having the given interval ratio r from the given note:

(a) $r = 5$ (b) $r = \frac{3}{5}$ (c) $r = 2.7$ (d) $r = \frac{2}{\pi}$

8. Express the following interval ratios in terms of n-chromatic units, for the given n. Round off to 2 digits to the right of the decimal.

   (a) ratio $\frac{7}{4}$; $n = 17$

   (b) ratio 3 ; $n = 7$

   (c) ratio 0.54 ; $n = 13$

   (d) ratio e ; $n = 3$ (i.e., major thirds)

9. For the values $n = 11, 19, 23$, find the n-chromatic scale's best approximation of the interval ratio $3/2$, and calculate the error in cents. Which of these values of n gives the best approximation, and is that approximation as good as that of the 12-chromatic scale?

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10. Analyze the basic harmony in the first five measures of *Moonlight Sonata*. Label the chords with appropriate suffix by root note class (e.g., IIm) and chord type (e.g., G^7). The music can be downloaded as a pdf file from the website.